

Local ergotropy across dissipation-driven quantum phase transitions in open Rabi models

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Abstract: We study open single- and two-qubit Rabi models, showing dissipation-driven BKT transitions, coherent-to-incoherent dynamical crossover, and ergotropy signatures of quantum criticality, linking environmental coupling, nonequilibrium relaxation, and work extraction in open quantum batteries.

We investigate the equilibrium and non-equilibrium properties of open Rabi systems, focusing on the single- and two-qubit models, where environmental interactions are mediated by a bosonic mode beyond the rotating-wave approximation. To address these problems, we combine tensor-network methods—Density-Matrix Renormalization Group (DMRG) for equilibrium and Time-Dependent Variational Principle (TDVP) for real-time dynamics—with worldline Monte Carlo simulations and analytical approaches inspired by Feynman and Mori. Within this framework, open quantum states are efficiently represented through Matrix Product States.

On the thermodynamic side, we show that increasing the system-environment coupling in the deep-strong-coupling regime drives a Berezinskii-Kosterlitz-Thouless (BKT) quantum phase transition in both the single- and two-qubit Rabi models [1,2]. On the dynamical side, shown in Fig.1 (a), we observe a coherent-to-incoherent crossover: at weak coupling the dynamics is characterized by Rabi oscillations, whereas stronger coupling leads to overdamped exponential relaxation, with relaxation functions flattening at the critical point [1].

As an application, we study work extraction from a subsystem regarded as an open quantum battery. In particular, we analyze local ergotropy and its fluctuations across the dissipative phase transition, and compare them with the corresponding switch-off protocol, where the energetic cost of decoupling the subsystem from the environment is explicitly included [3]. Our results show that both the static and dynamical behavior of ergotropy provide clear signatures of the BKT transition (see Fig.1 (b)), highlighting a direct connection between dissipation-driven quantum criticality and the performance of open quantum batteries.

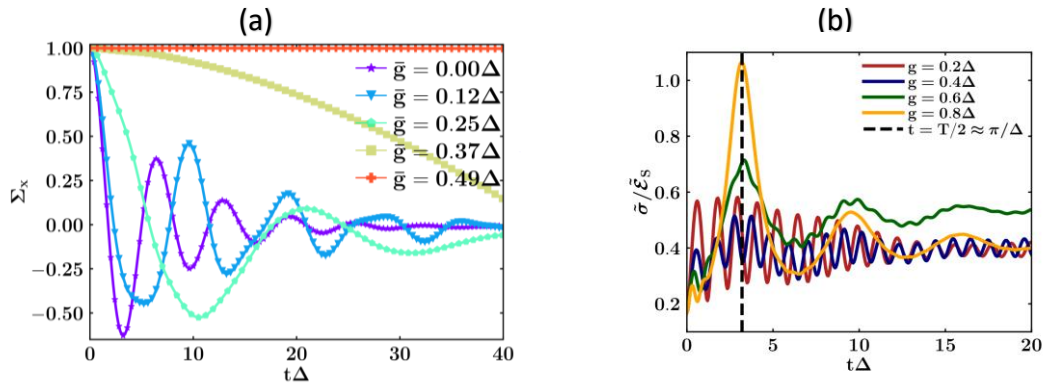


Fig. 1. (a) Oscillator relaxation function $\Sigma_x(t\Delta)$ for different coupling values $\bar{g} \in [0.00, 0.59]\Delta$ (shown in rainbow colors from violet to red) near the critical point, computed using TDVP. (b) Relative fluctuations of the lower bound of local ergotropy (solid lines) as a function of dimensionless time for increasing system-environment coupling g/Δ (from red to orange). The vertical dashed black line is the time when the bath is in counterphase, farthest from initial state.

References:

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