

Graph Similarity With Bipartite Gaussian Boson Sampling in Time-Frequency Modes

M. Borghi,^{1,*} E. Brusaschi,¹ M. Liscidini,¹ M. Galli,¹ and D. Bajoni²

¹ Dipartimento di Fisica, Università di Pavia, Via Agostino Bassi 6, 27100 Pavia, Italy

² Dipartimento di Ingegneria Industriale e dell'Informazione, Università di Pavia, Via Adolfo Ferrata 5, 27100 Pavia, Italy

Abstract: We demonstrate a bipartite gaussian boson sampling experiment with four photons in six frequency-time bin modes, using the device to assess similarity between different graph families of six vertices. © 2026 The Author(s)

Multi-photon quantum interference forms the backbone of scalable quantum computing architectures and communication networks. Boson-sampling problems, specific instances of multi-photon interference, have fueled interest in near-term quantum photonic devices that may outperform classical computers for specialized tasks. In these experiments, non-classical light is fed into a linear interferometer, and multi-photon patterns are sampled at the output. Simulating these distributions is computationally challenging, as they are tied to matrix functions like the permanent or Hafnian, which lack efficient classical algorithms [1]. When squeezed states are used in combination with linear interferometers, the output samples can be implemented in graph-search algorithms, and for the calculation of the vibronic spectra of molecules [2]. While previous GBS experiments relied on path or time-bin encoding of optical modes, frequency-bin encoding -despite its inherently high dimensionality and potential for parallel operations in a single spatial mode - remains largely unexplored. Here we report the realization of a GBS experiment utilizing up to four photons in six time-frequency-bin modes [3]. Two-mode squeezed light is generated by an integrated silicon nitride microresonator by spontaneous four-wave mixing at not degenerate wavelengths and in two time-bins. These modes are interfered using an unbalanced interferometer with electro-optic phase modulators that scramble their frequencies. We recorded 144 different four-photon detection patterns at the output and reconstructed their probabilities, finding close agreement with simulations. The output samples are used to construct feature vectors that are exploited to cluster graphs based on their similarity.

In the experimental setup a pump laser delivers two consecutive rectangular optical pulses at 1544.53 nm. each lasting $\tau = 800$ ps and separated by $\tau = 20$ ns, generating two-mode squeezed light on a pair of signal and idler resonances by spontaneous four-wave mixing and in two temporal modes (early (E) and late (L)). From the measured low-power joint temporal intensity which we assess nearly single temporal mode emission [3]. Light at the output of the chip is fed to a fiber-based Franson interferometer, in which an electro-optic phase modulator (PM) is inserted in both arms. The unbalanced interferometer mixes the temporal modes, while the PMs, driven at 18.5 with a modulation index of $\delta \sim 1.4$, scramble their frequency into three dominant modes. The relative phase x of the two arms of the interferometer is stabilized to a controllable set-point. One output is frequency-demultiplexed using a programmable waveshaper and directed to four superconducting nanowire single-photon detectors. This experiment maps to an instance of bipartite GBS, where the probability to observe a multi-photon output pattern in a specific set of modes is determined by the Hafnian of a matrix U , which depends on the interferometer and the input squeezed modes [4]. Since the Hafnian is linked to perfect matchings in graphs [1], U can be interpreted as the complex adjacency matrix of a bipartite graph, with signal and idler modes forming the two vertex sets. The entries of the matrix U can be reconfigured by changing the phase x , the modulation index δ and the relative phase Δ between the RF signals driving the PMs.

Building on Refs. [2, 4], we construct orbit probabilities from the detected photon pattern probabilities, assembling them into feature vectors. The distance between feature vectors is used to quantify the similarity of the corresponding graphs encoded in the GBS experiments. Our device encodes a 12-vertex graph, from which we derive 12 families of six-vertex subgraphs. Graphs belonging to the same family are isomorphic, but the different families can not be mapped to each other by a permutation of their vertices. We measured two and four photons orbits $O(1, 1)$ and $O(1, 1, 1, 1)$, and use them to correctly build the feature vector. We show that isomorphic graphs cluster together within the same family, while distinct families form separate clusters.

References

1. R. Kruse, et. al., *Phys. Rev. A*, vol. 100, no. 3, p. 032326, 2019.
2. T. R. Bromley, et. al., *Quantum Sci. Technol.*, vol. 5, no. 3, p. 034010, 2020.
3. B. Massimo, et al., *arXiv preprint arXiv:2502.00491* (2025).
4. J. M. Arrazola, et. al., *Nature*, vol. 591, no. 7848, pp. 54–60, 2021.