Fundamental precision bounds for noisy quantum metrology: purification-based methods and their applications

Francesco Albarelli[1,2]

1. *Università di Parma, Dipartimento di Scienze Matematiche, Fisiche e Informatiche, I-43124 Parma, Italy*
2. *INFN—Sezione di Milano-Bicocca, Gruppo Collegato di Parma, I-43124 Parma, Italy*

Abstract: Purification-based methods provide a unified route to fundamental precision bounds in noisy quantum metrology. I review their logic, recent extensions to correlated dynamics, and applications to continuous monitoring, dephasing estimation, and time-dependent signal sensing problems.

Quantum metrology seeks the ultimate precision limits for parameter estimation using quantum resources. In realistic settings, noise strongly constrains the achievable advantage, making the derivation of reliable and computable precision bounds a central problem. In this contribution I will present an overview of purification-based bounds, which provide a conceptually transparent and versatile framework to upper bound the quantum Fisher information in noisy estimation problems. These methods recast the metrological problem in an enlarged Hilbert space, where optimization over equivalent purifications yields bounds that are often both physically insightful and numerically tractable. Besides identifying fundamental limitations, they also naturally raise the question of how such bounds may actually be saturated in practice.

I will first discuss the role of purification-based techniques in the modern theory of noisy quantum metrology, emphasizing how they connect channel-based descriptions, adaptive protocols, and asymptotic precision limits. In particular, I will briefly review recent results on fundamental bounds for general adaptive metrological schemes [1] and on their extension to temporally and spatially correlated noise [2], where the estimation problem is naturally formulated in terms of quantum combs. These developments significantly enlarge the scope of noisy quantum metrology beyond the standard independent-channel setting and provide a systematic route to identifying fundamental limitations in complex sensing scenarios. In parallel, a major line of research has shown that, in suitable time-independent settings, the ultimate bounds can in principle be attained through suitably designed quantum error-correction strategies.

I will then focus on applications. A first example concerns noisy bosonic metrology, where recent work has clarified the interplay between available energy, interrogation time, and noise, providing useful insight into optimal resource allocation in open-system sensing problems [3]. A second application concerns continuously monitored quantum systems, where purification-based bounds make it possible to quantify in a transparent way the impact of inefficient detection on the achievable precision, as highlighted in the context of continuously monitored dissipative many-body dynamics [4]. At the same time, they can also show that in relevant regimes comparatively simple semiclassical measurement and estimation strategies are already optimal, without the need for more complicated protocols [3]. These examples also illustrate the usefulness of applying the logic of fundamental bounds to continuously monitored settings, beyond more conventional metrological scenarios.

Finally, I will outline ongoing directions motivated by these techniques. The first concerns collective dephasing estimation, where recent results on nonequilibrium pure-dephasing thermometry [5] and on stochastic signal estimation under correlated fluctuations [6] motivate a more systematic analysis of metrological gains and limitations, with particular focus on spin-based platforms. The second concerns time-dependent signal estimation in noisy settings, with the goal of extending control-enhanced metrological methods developed for noiseless time-dependent Hamiltonians [7] to a framework that treats decoherence on equal footing, in a more general and systematic way than earlier approaches [8]. From this perspective, a key open question is whether the ultimate purification-based bounds can also be attained in these more general scenarios, and in particular how error-corrected strategies may be generalized from the standard time-independent case to the estimation of time-dependent signals.

Overall, the talk aims to show that purification-based bounds are not only a technical tool for deriving no-go theorems, but also a unifying perspective for identifying physically meaningful limits, comparing sensing strategies, and guiding the analysis of emerging metrological scenarios involving correlated noise, continuous measurements, and time-dependent signals.

NQSTI National Quantum Science and Technology Institute

References

[1] Stanisław Kurdziałek, Wojciech Górecki, Francesco Albarelli, and Rafał Demkowicz-Dobrzański, "Using Adaptiveness and Causal Superpositions Against Noise in Quantum Metrology," Physical Review Letters 131, 090801 (2023).
[2] Stanisław Kurdziałek, Francesco Albarelli, and Rafał Demkowicz-Dobrzański, "Universal Bounds for Quantum Metrology in the Presence of Correlated Noise," Physical Review Letters 135, 130801 (2025).
[3] Wojciech Górecki, Francesco Albarelli, Simone Felicetti, Roberto Di Candia, and Lorenzo Maccone, "Interplay Between Time and Energy in Bosonic Noisy Quantum Metrology," PRX Quantum 6, 020351 (2025).
[4] Eoin O'Connor, Victor Montenegro, Francesco Albarelli, Matteo G. A. Paris, Abolfazl Bayat, and Marco G. Genoni, "Attaining Quantum Sensing Enhancement from Monitored Dissipative Time Crystals," arXiv:2508.15448 (2025).
[5] Francesco Albarelli, Matteo G. A. Paris, Bassano Vacchini, and Andrea Smirne, "Invasiveness of nonequilibrium pure-dephasing quantum thermometry," Physical Review A 108, 062421 (2023).
[6] Anirban Dey, Sara Mouradian, Cosmo Lupo, and Paola Cappellaro, "Quantum-Optimal Frequency Estimation of Stochastic ac Fields," Physical Review Letters 135, 130802 (2025).
[7] Shengshi Pang and Andrew N. Jordan, "Optimal adaptive control for quantum metrology with time-dependent Hamiltonians," Nature Communications 8, 14695 (2017).
[8] Tuvia Gefen, Fedor Jelezko, and Alex Retzker, "Control methods for improved Fisher information with quantum sensing," Physical Review A 96, 032310 (2017).