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Dynamical entanglement percolation with spatially correlated disorder

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Abstract: We study dynamical entanglement percolation in qubit networks with spatially correlated disorder, revealing hysteretic behavior absent in the uncorrelated case, explained through a two-color bond percolation model.

Quantum networks are central to distributed quantum information [1–3]. A key challenge is establishing long-range entanglement. Entanglement percolation models [4,5] study when a giant cluster of maximally entangled pairs spans the network: each edge carries an entangled pair convertible to a singlet with probability p, and above a critical threshold entanglement percolates.

In this work [6], we extend this framework to a dynamical setting where entanglement is not simply distributed but generated by direct qubit–qubit interactions. We consider a network of qubits evolving under pairwise σx interactions with coupling frequencies ω_e. Starting from a product state, each edge develops a time-dependent entanglement quantified by a singlet conversion probability $\varphi_e(t) = 1 - |\cos(\omega t)|$. We then analyse the resulting percolation properties of the network as a function of time.

For uncorrelated disorder, where the frequencies ω_e are independent random variables, we prove that at long times the average fraction of active edges converges to $p\infty = 1 - 2/\pi \approx 0.363$ for any continuous distribution. Crucially, P(p) remains identical to standard uniform bond percolation: the dynamical evolution simply moves the system along the static curve $P_0(p)$.

The picture changes qualitatively when spatial correlations are introduced. We assign frequencies as deterministic functions of inter-node distances in a geometrically perturbed lattice, so that edges sharing a node acquire correlated frequencies. Numerical simulations reveal that P(p) becomes multi-valued: the network exhibits hysteresis, oscillating between percolating and non-percolating phases along distinct branches (Fig. 1(c)–(d)).

To elucidate this phenomenon, we map the dynamics onto a two-colour inhomogeneous bond percolation model on a square lattice with alternating red/blue edge patterns. Each colour is activated with a different probability, and the spatial constraint produces large-scale structural patterns absent in randomly reshuffled configurations. The phase diagram $S(\varphi_1, \varphi_2)$, shown in Fig. 1(a)–(b), is determined via extensive simulations and a mean-field theory. Hysteresis arises from non-monotonic time evolution combined with strong spatial constraints, revealing percolation phenomenology significantly richer than standard entanglement percolation.

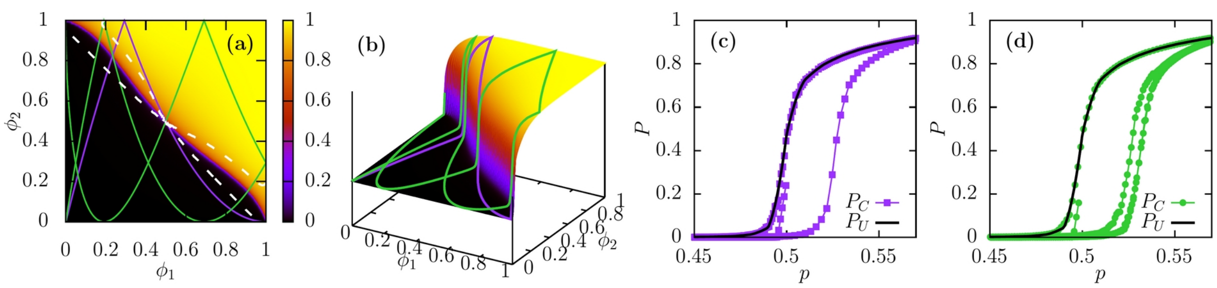

Fig. 1. Phase diagram of two-colour bond percolation on a square lattice. (a) Order parameter in the φ1–φ2 plane; continuous lines show dynamical evolutions for Ω = 2 (purple) and Ω = 5/2 (green). (b) 3D surface S(φ1, φ2). (c)–(d) Parametric plots of P(p) showing hysteretic multi-branch behaviour (symbols) versus uniform bond percolation (black line).

Our results demonstrate that even simple interaction-driven quantum networks exhibit percolation phenomenology significantly richer than standard entanglement percolation, with potential implications for the design and analysis of realistic quantum network architectures.

References

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