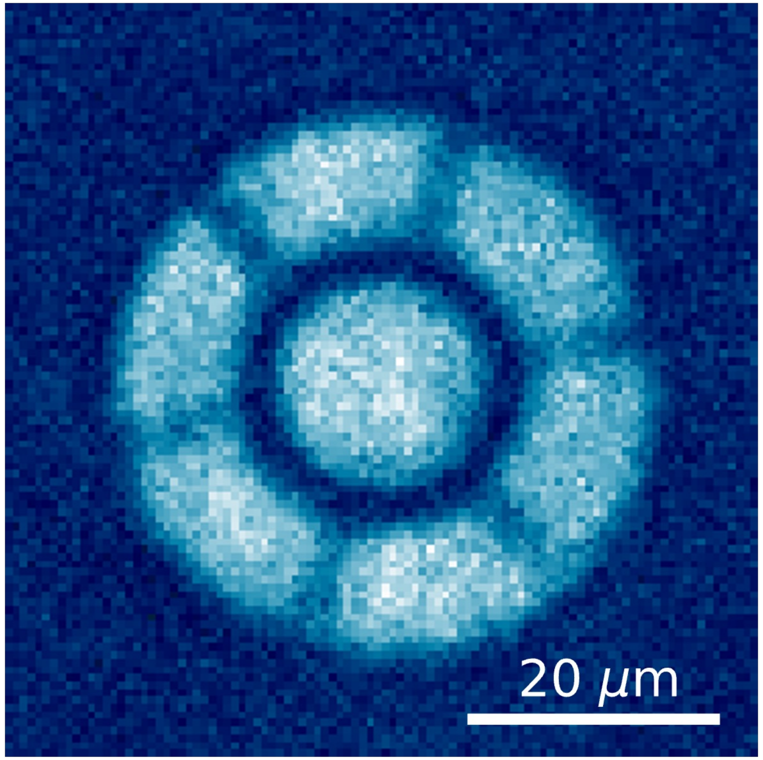
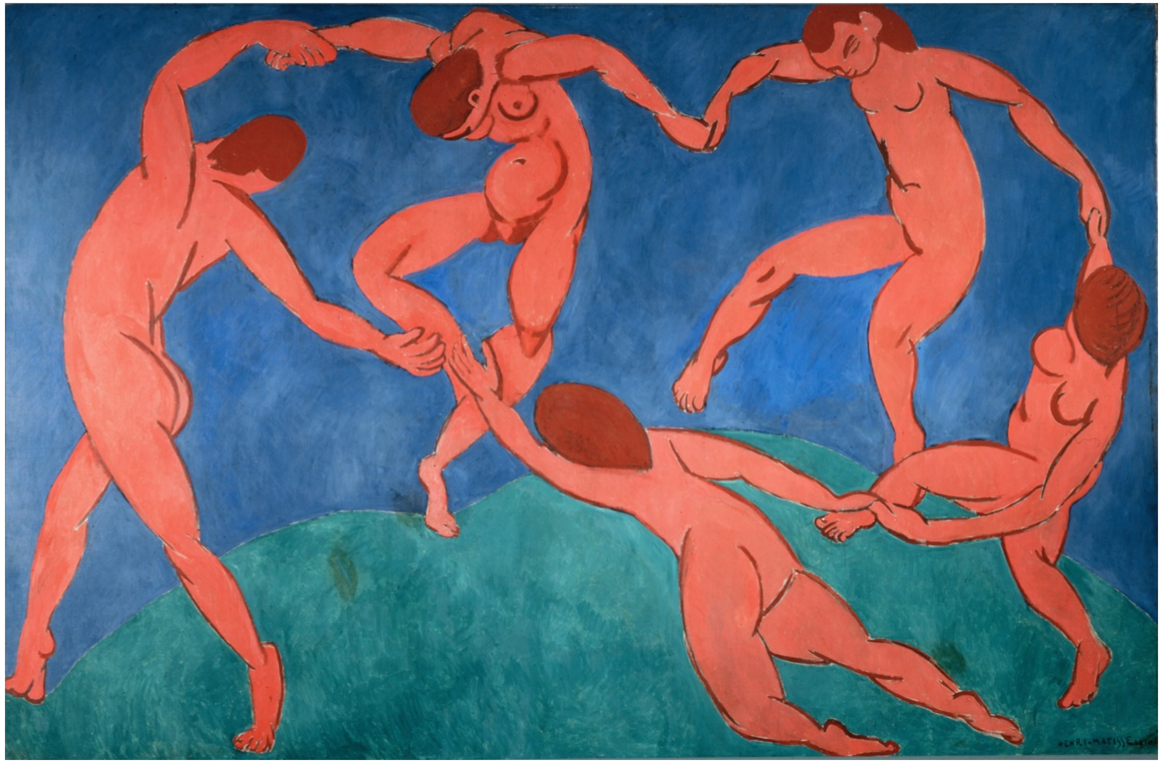
STABILIZING PERSISTENT CURRENTS IN A JOSEPHSON JUNCTION NECKLACE

we study metastable finite circulation states in a toroidal superfluid with a variable number, n, of Josephson junction (Josephson junction necklace)

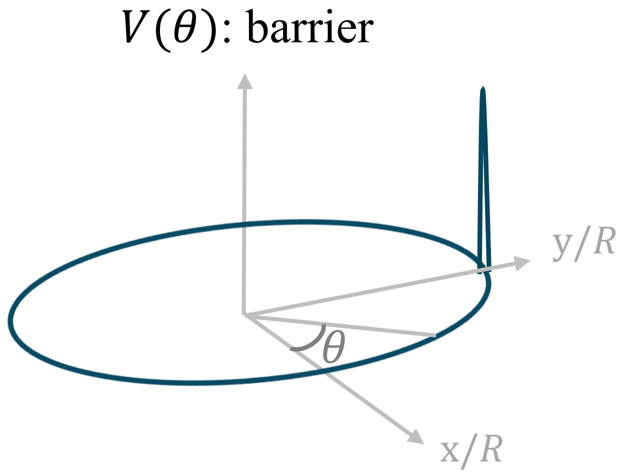


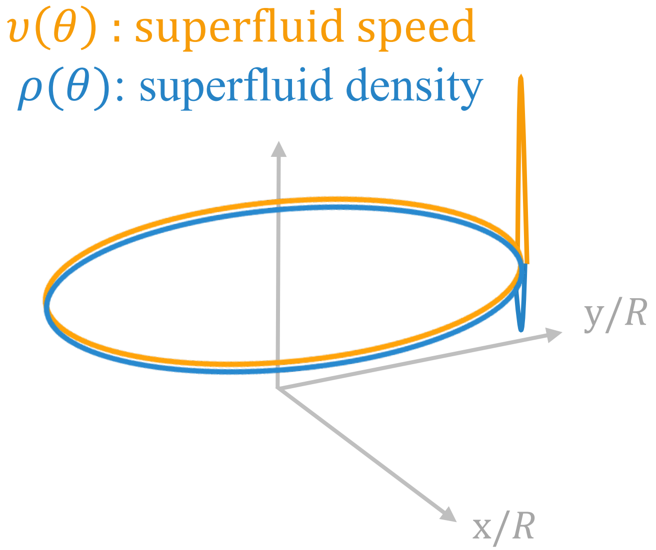


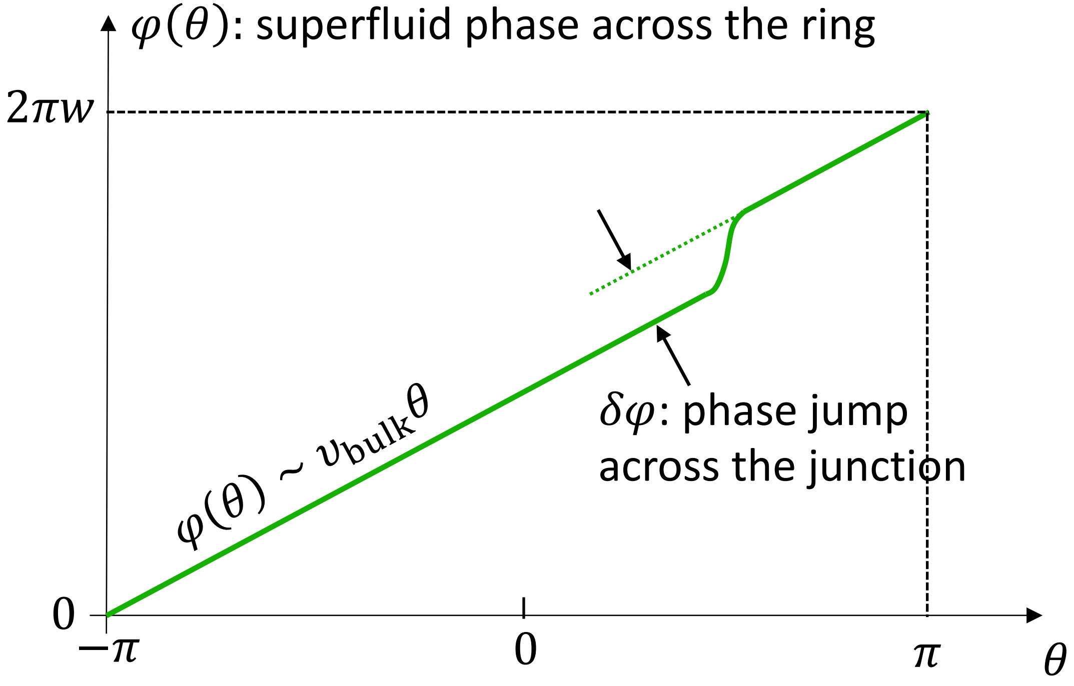
the n junctions are spatially-separated but are *not independent* because of the system's topology

The physical explanation is easier in 1D with narrow junctions:

the current is constant along the ring, $J = \rho(\theta)v(\theta)/R = 2\pi w / \int_0^{2\pi} d\theta/\rho(\theta)$







$$v(\theta) = \frac{\hbar}{mR}\frac{d\varphi}{d\theta}$$

$$\varphi(+\pi) - \varphi(-\pi) = 2\pi v_{\text{bulk}} + \delta\varphi = 2\pi w$$

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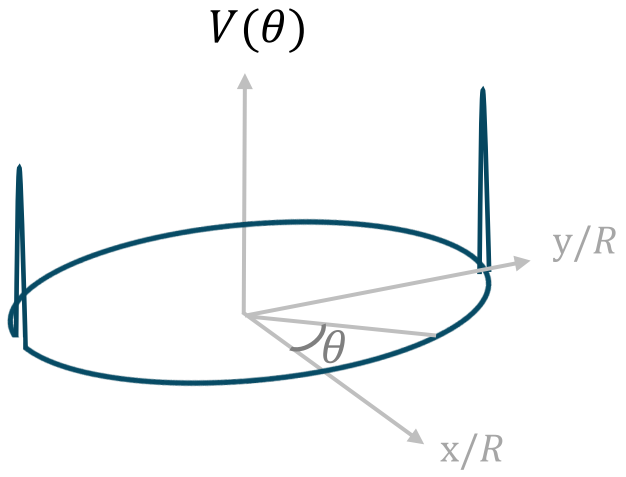

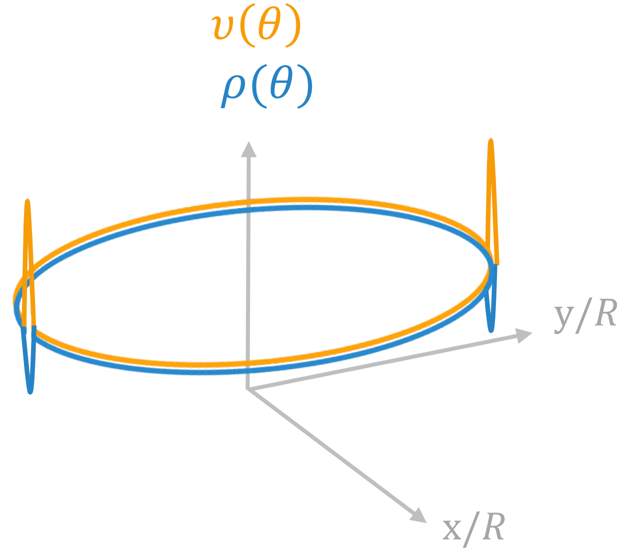

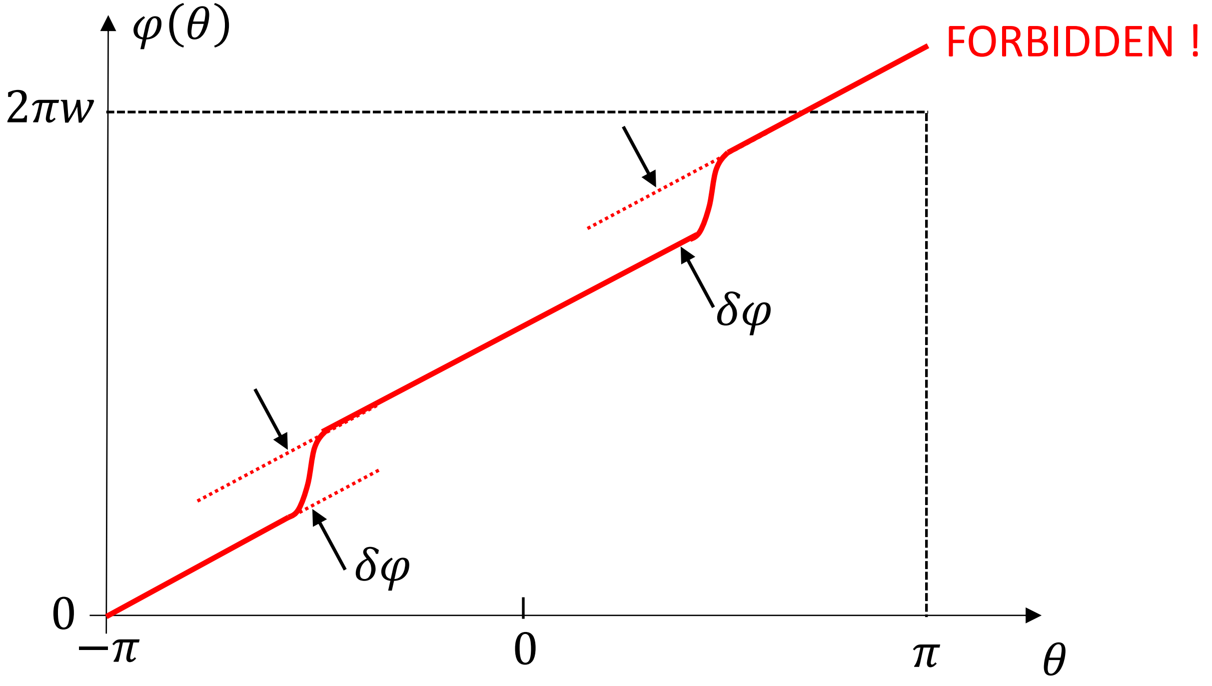

J, v_{bulk} , ρ_{bulk} : expected independent from the number of junctions

$\delta\varphi$ must be a function of the number of junctions

$\varphi(+\pi) - \varphi(-\pi) = 2\pi v_{\mathrm{bulk}} + n\delta\varphi = 2\pi w$

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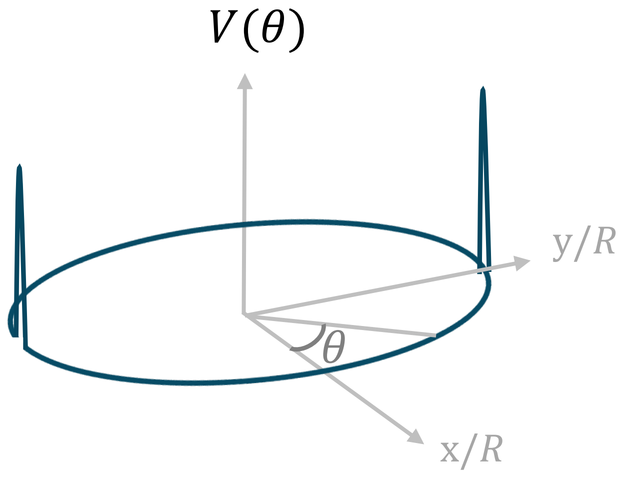

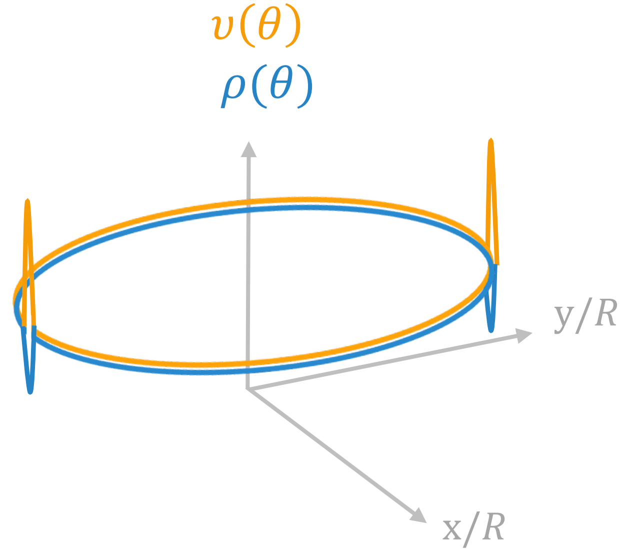

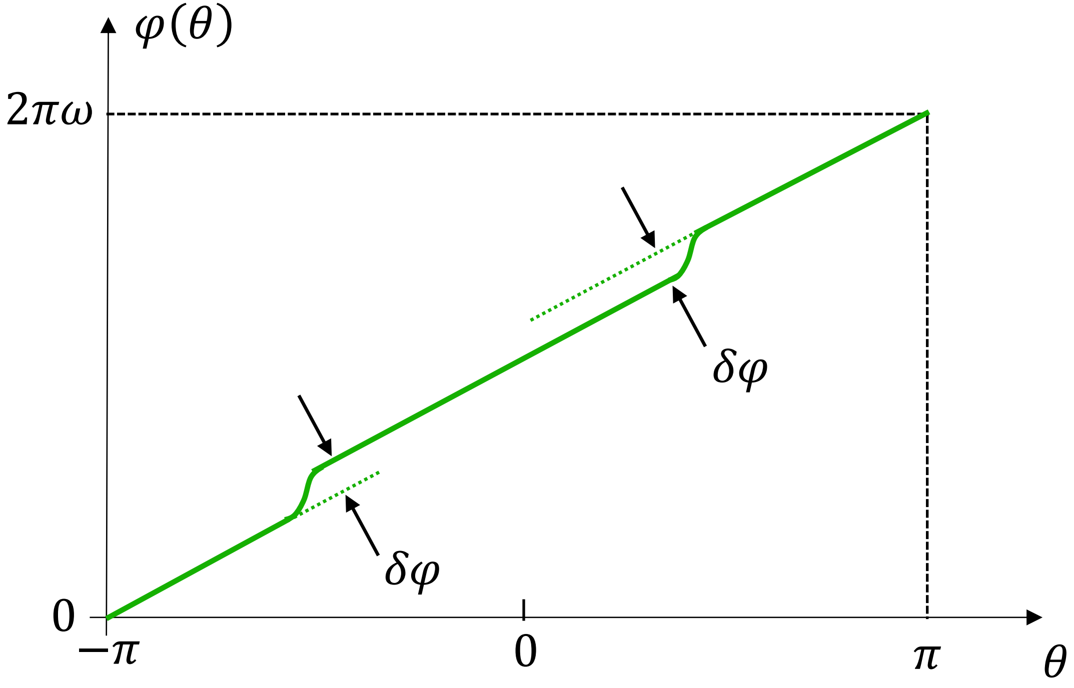

$J, v_{\text{bulk}}, \rho_{\text{bulk}}$: expected independent from the number of junctions

$\delta\varphi$ must be a function of the number of junctions: $\delta\varphi(n) \sim 1/n$

$$\varphi(+\pi) - \varphi(-\pi) = 2\pi v_{\text{bulk}} + n\delta\varphi(n) = 2\pi w$$

The physical explanation is easier in 1D with narrow junctions:

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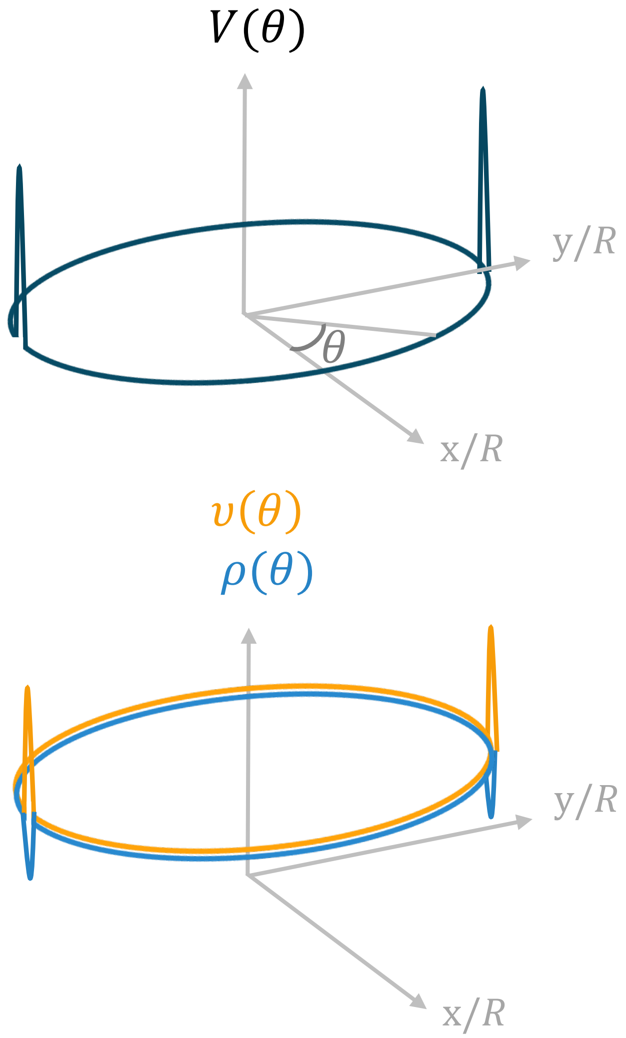
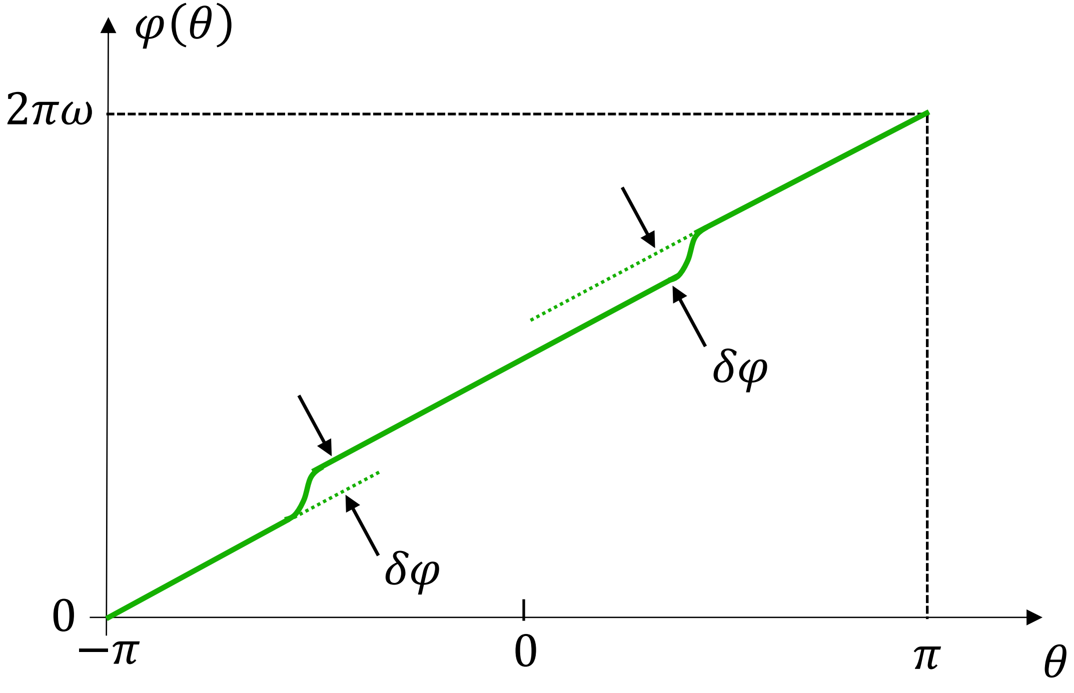




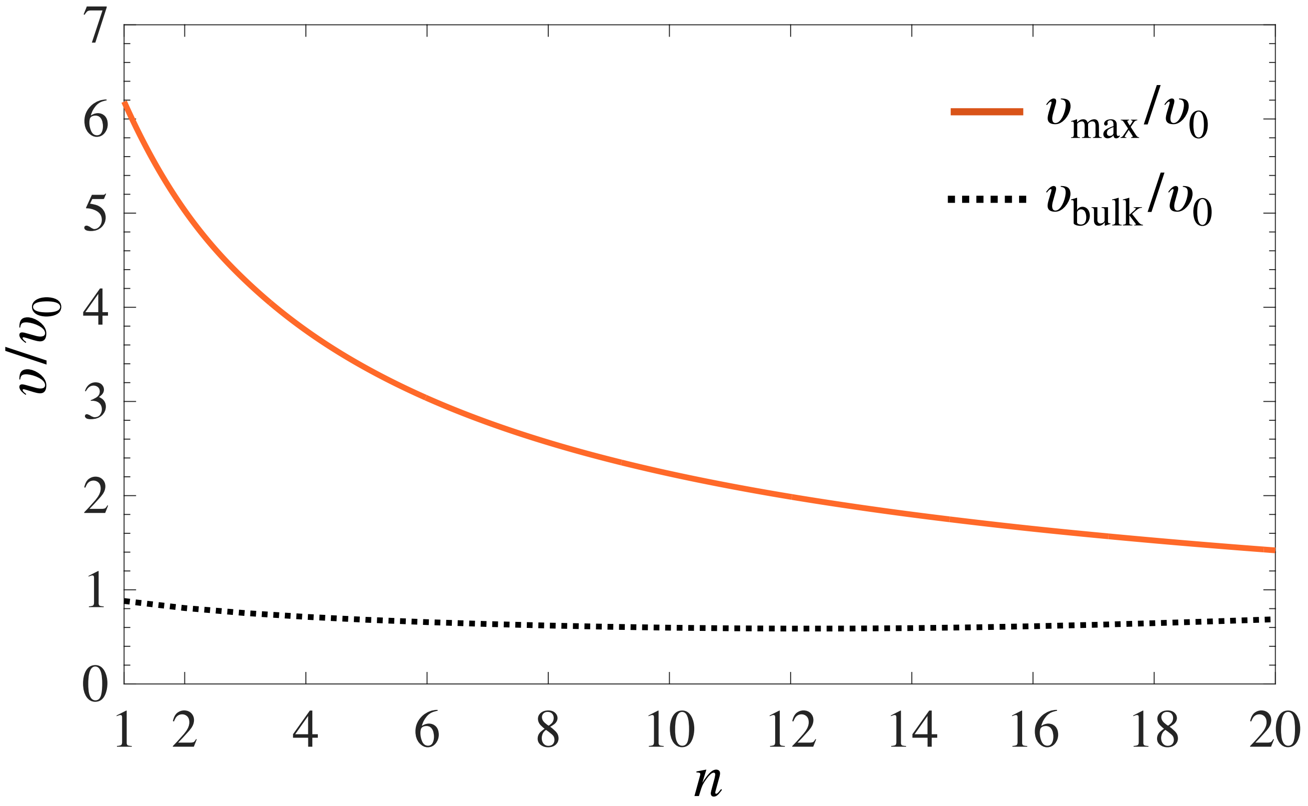
$\delta\varphi(n) \sim 1/n$

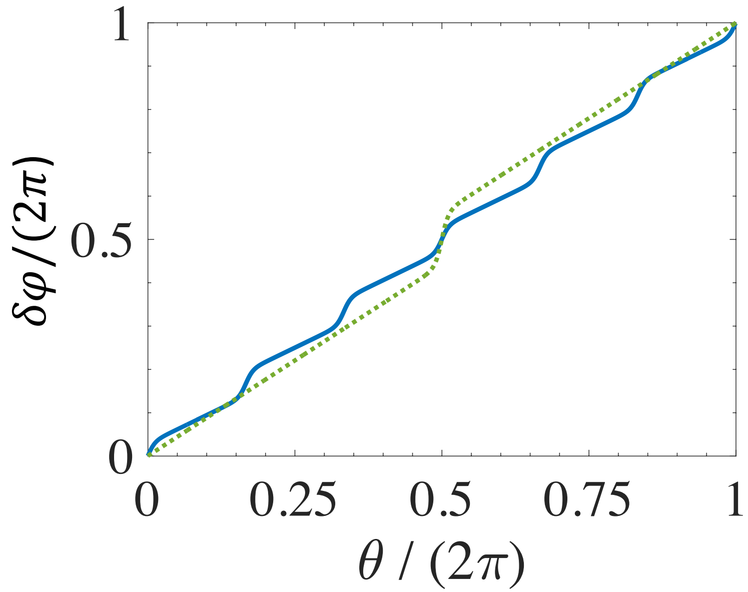
$J = J_c(n) \sin \delta\varphi(n) \sim J_c(n)\delta\varphi(n)$ ⇨ $J_c(n) \sim n$

1D beyond the approximation of narrow junctions

$$\delta\varphi \approx \frac{2\pi w}{n} \qquad J_c \approx \frac{\hbar}{mR^2}\frac{n f(w,n)}{8\pi} \qquad f(w,n) \stackrel{\text{def}}{=} (2\pi)^2 \left[\int_0^{2\pi} d\theta \, \frac{1}{\rho(\theta; w, n)}\right]^{-1} \leq f_s$$

numerical solutions of the 1D GPE:





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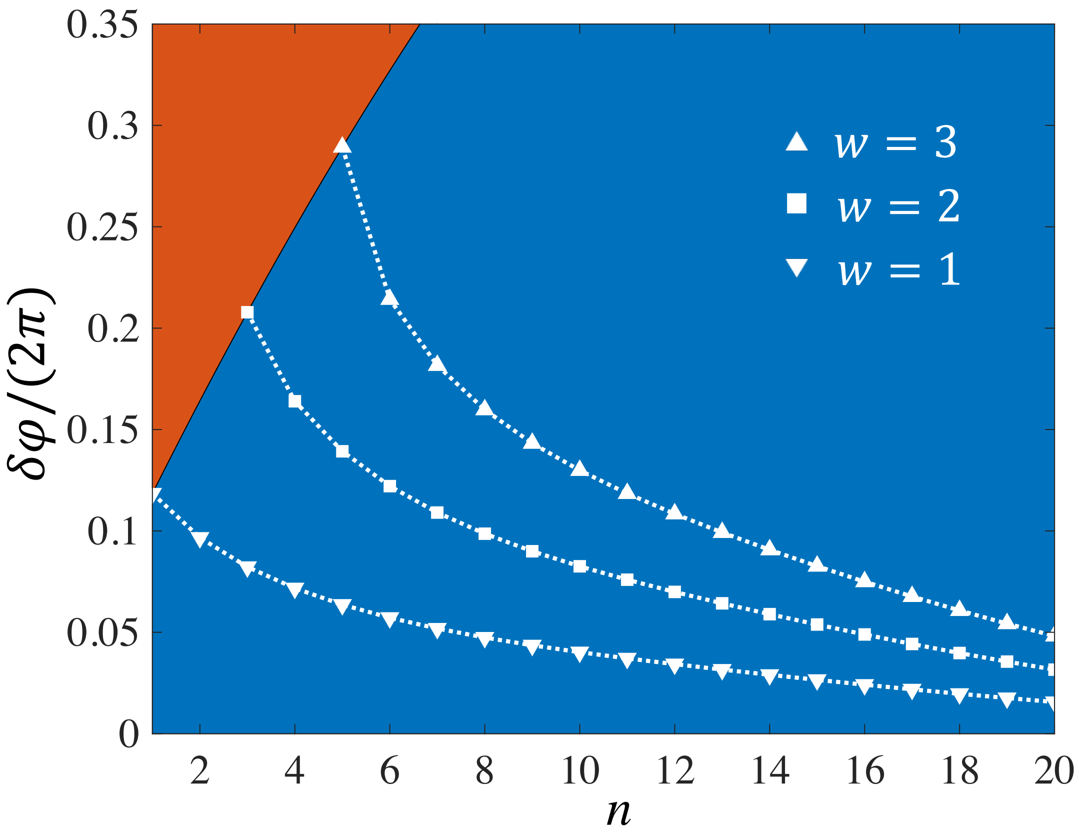

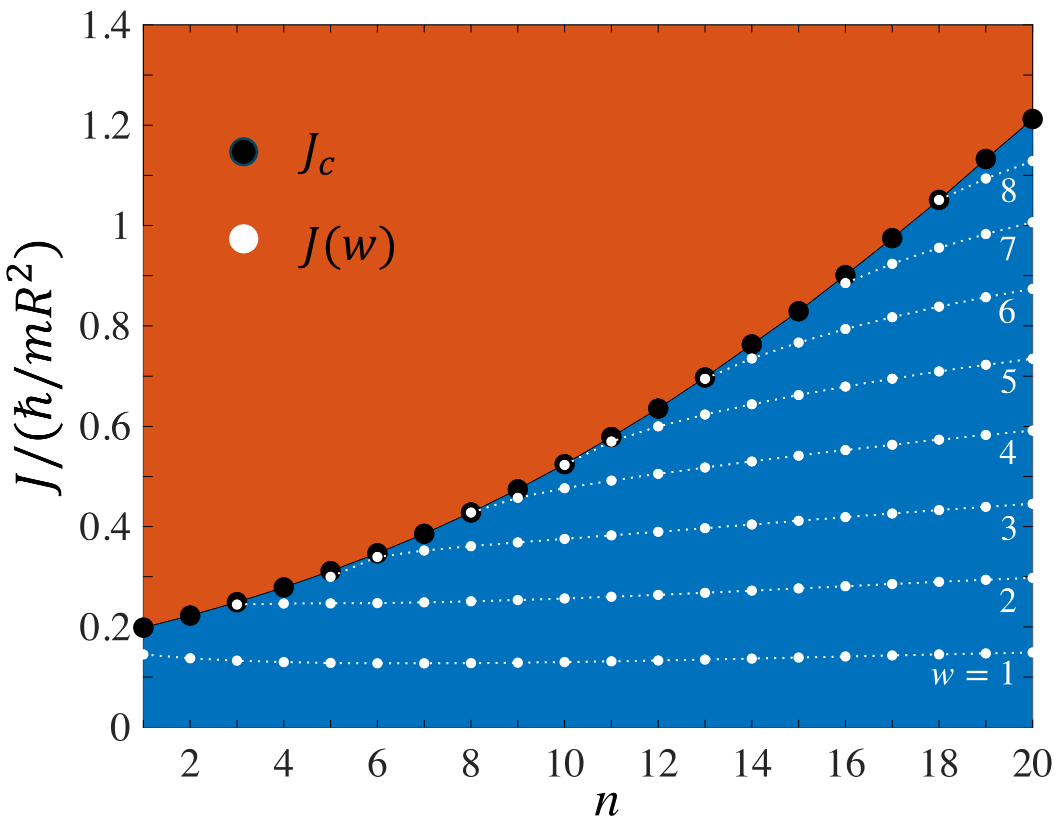

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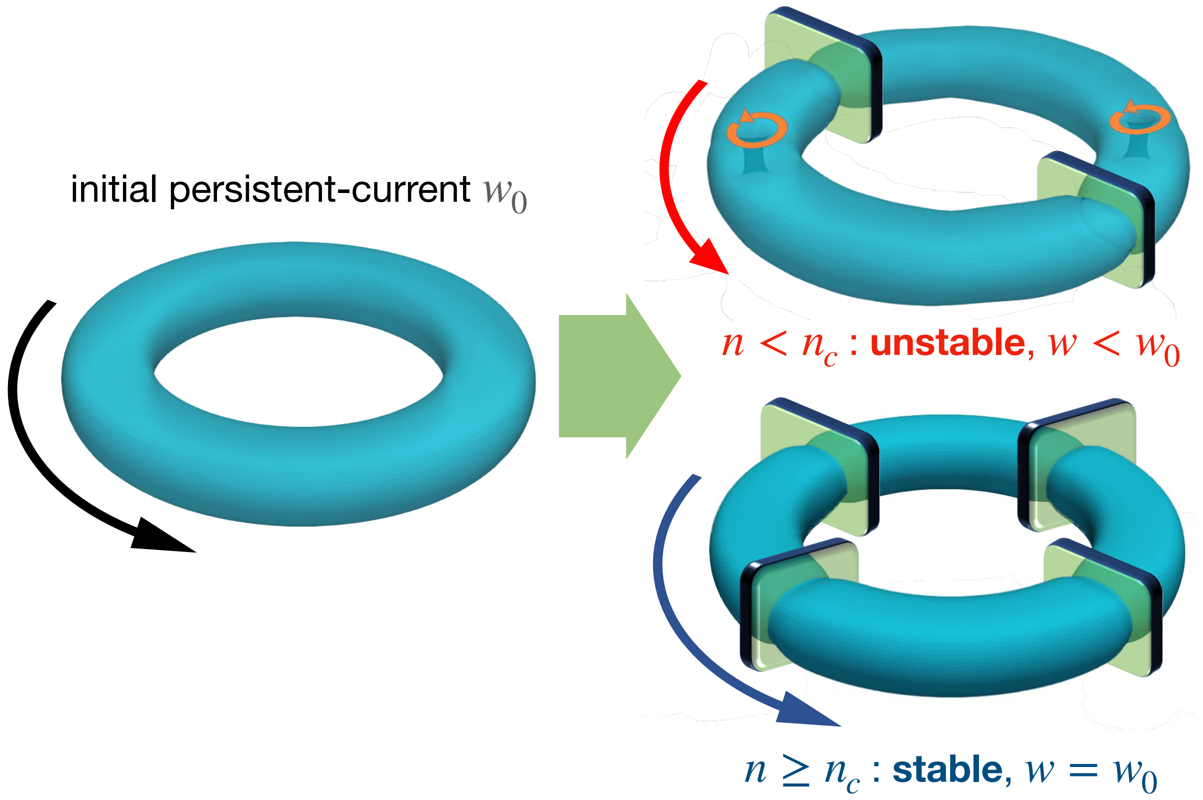
Experiment

- Preparation of finite circulation states in the clean ring $w_0 = [1,2,3,4]$
- Ramp up the barriers over $1\ ms$ (larger than $\hbar/\mu$ and shorter than typical dynamics times)



$R_{in} \approx 11\ \mu m$
$R_{out} \approx 21\ \mu m$
$\mu/(2\pi\hbar) = 850\ Hz$
$V_0/\mu = 1.3$
$\sigma/\xi = 1.2$

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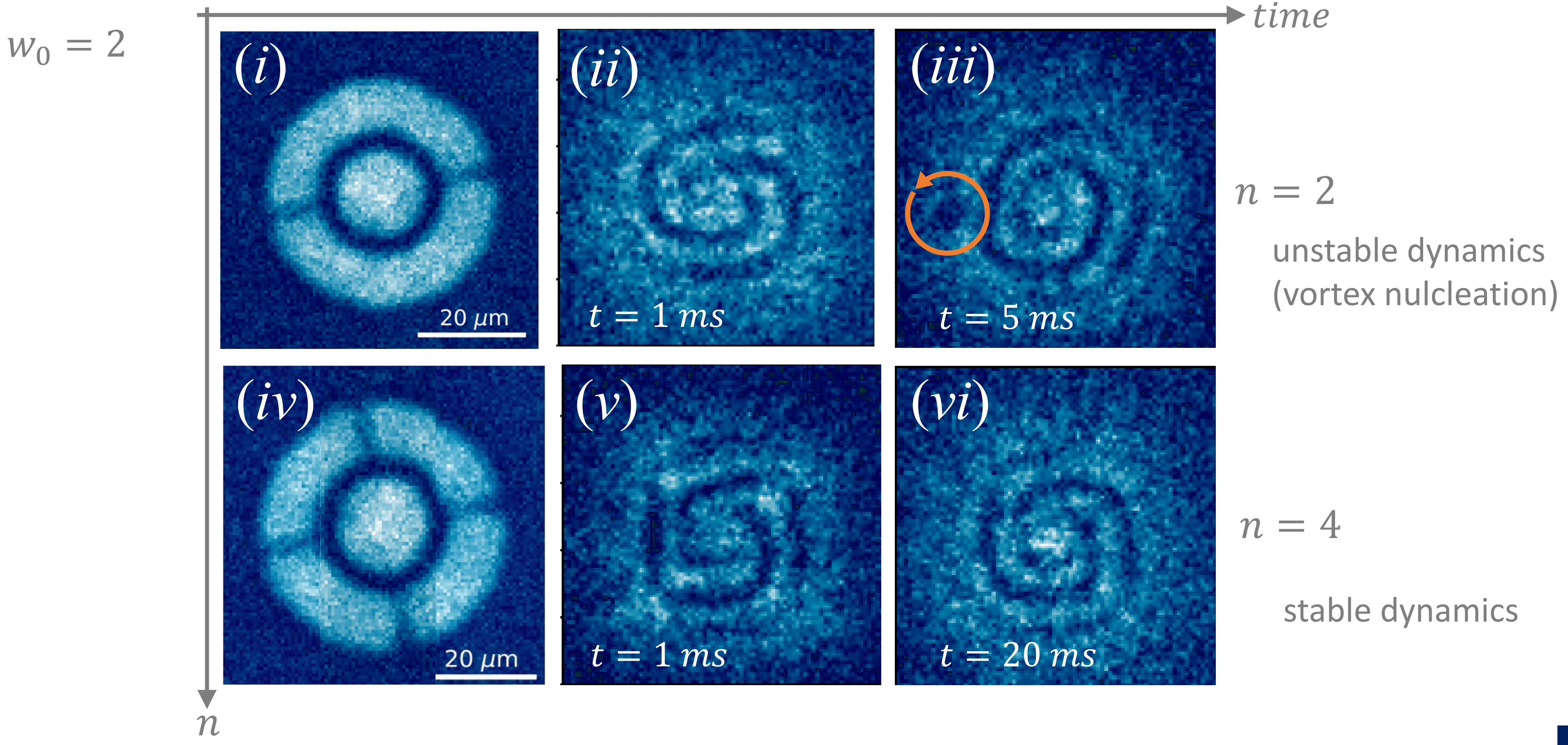
Single shots interferograms



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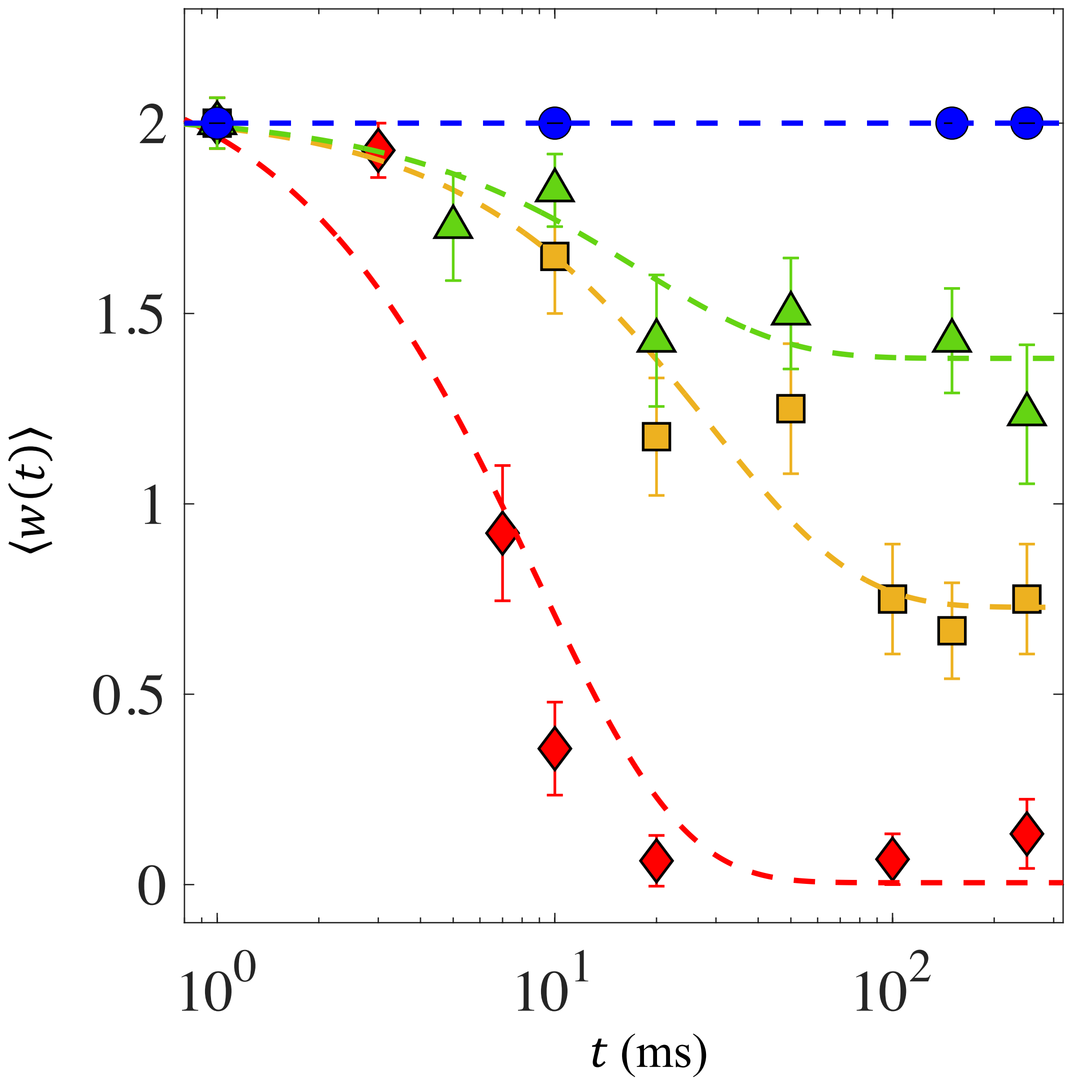
Average circulation and stability



$$\langle w(t) \rangle = w_f + \Delta\, w\, e^{-\Gamma t}$$

each point is the average over 15 realizations

$w_0 = 2$

◆ $n = 2$

■ $n = 4$

▲ $n = 6$

● $n = 8$

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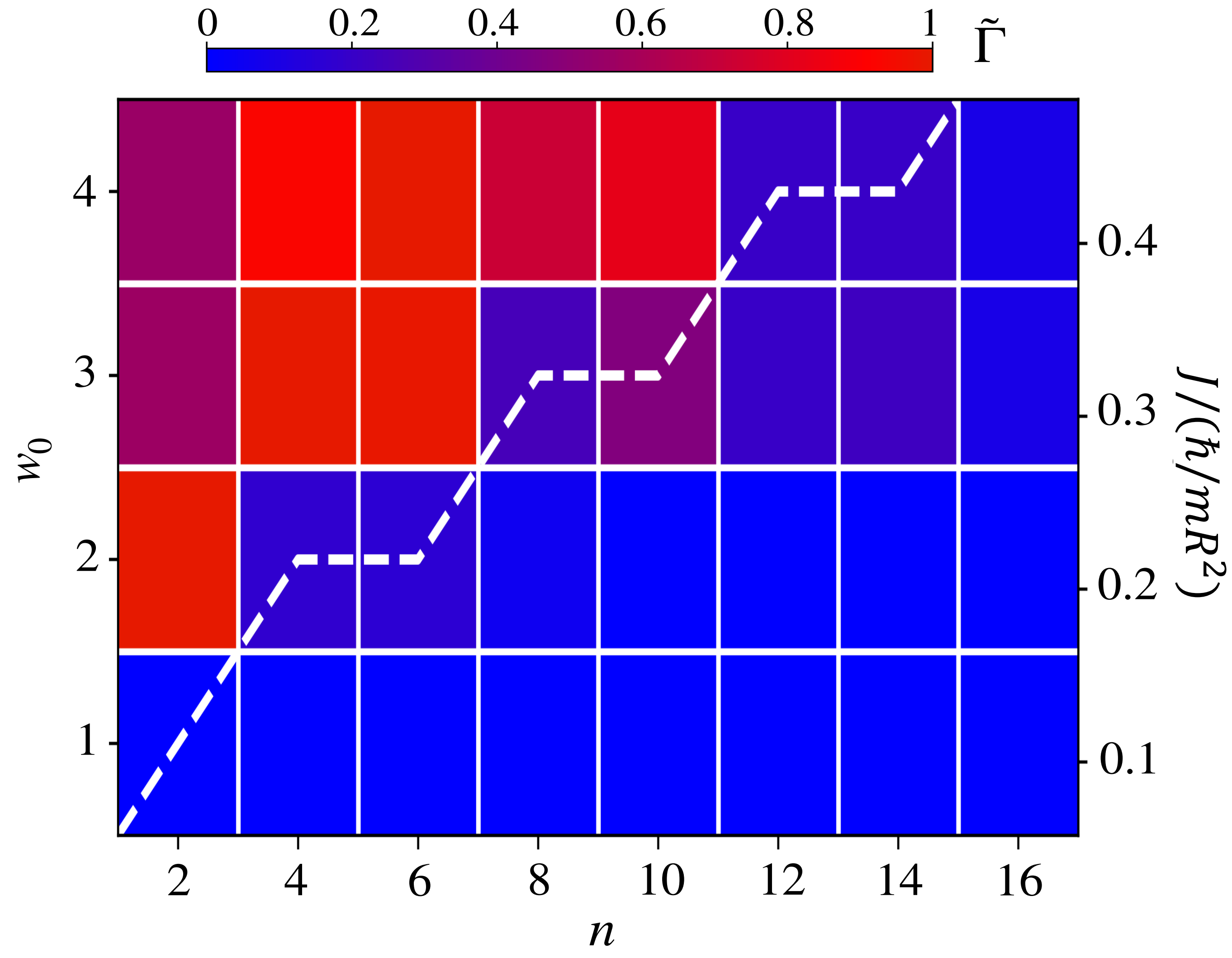
Average circulation and stability



color map: experiment

$$\tilde{\Gamma} = \frac{\Delta w \, \Gamma(n, w_0)}{\max_n \Delta w \, \Gamma(n, w_0)}$$

statistical information:

most of the trajectories $w(t)$ have dacayed in time

most of the trajectories are stable: $w(t) = w_0$

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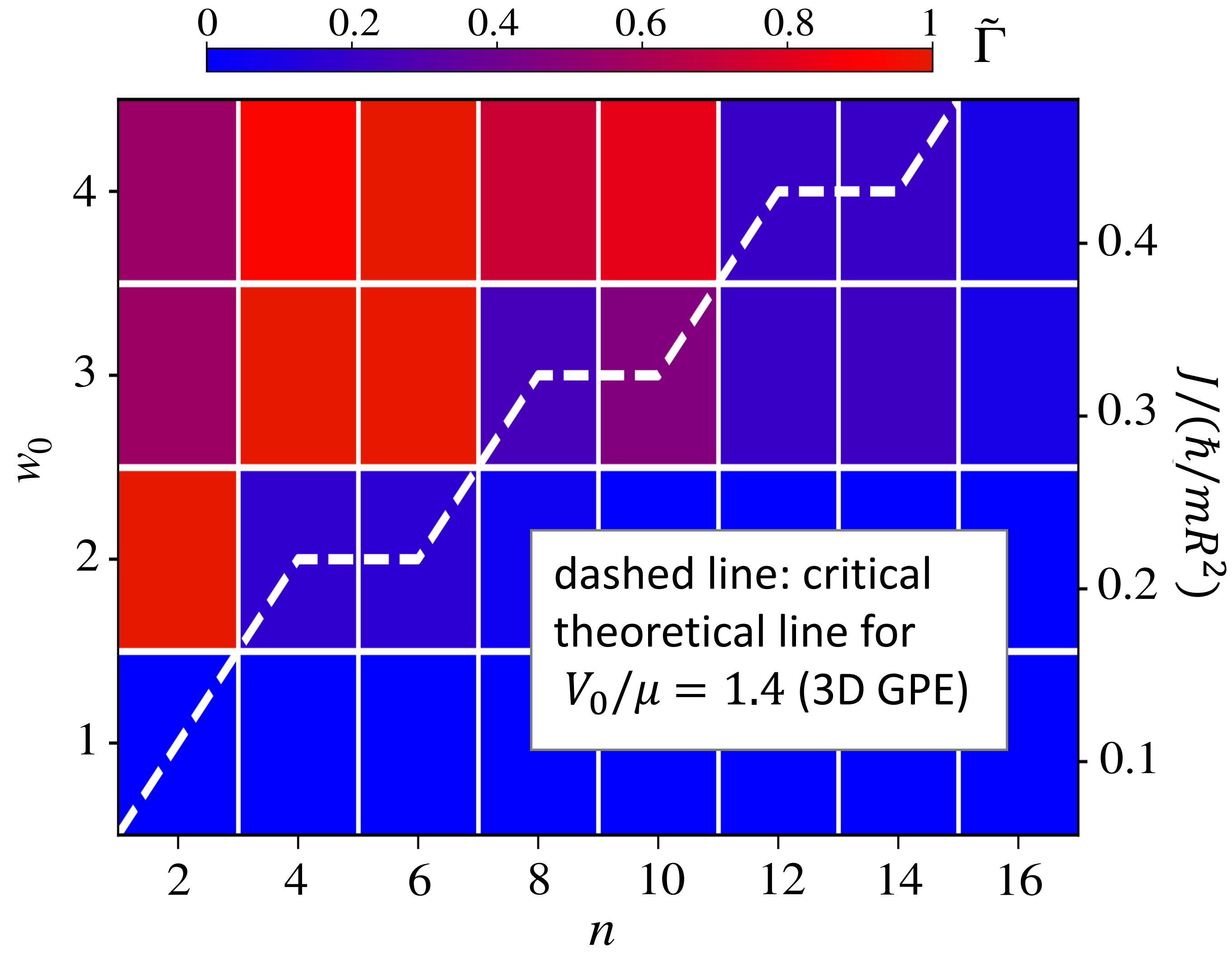
Average circulation and stability



color map: experiment

$$\tilde{\Gamma} = \frac{\Delta w\, \Gamma(n, w_0)}{\max_n \Delta w\, \Gamma(n, w_0)}$$

statistical information:

Red: most of the trajectories $w(t)$ have dacayed in time

Blue: most of the trajectories are stable: $w(t) = w_0$

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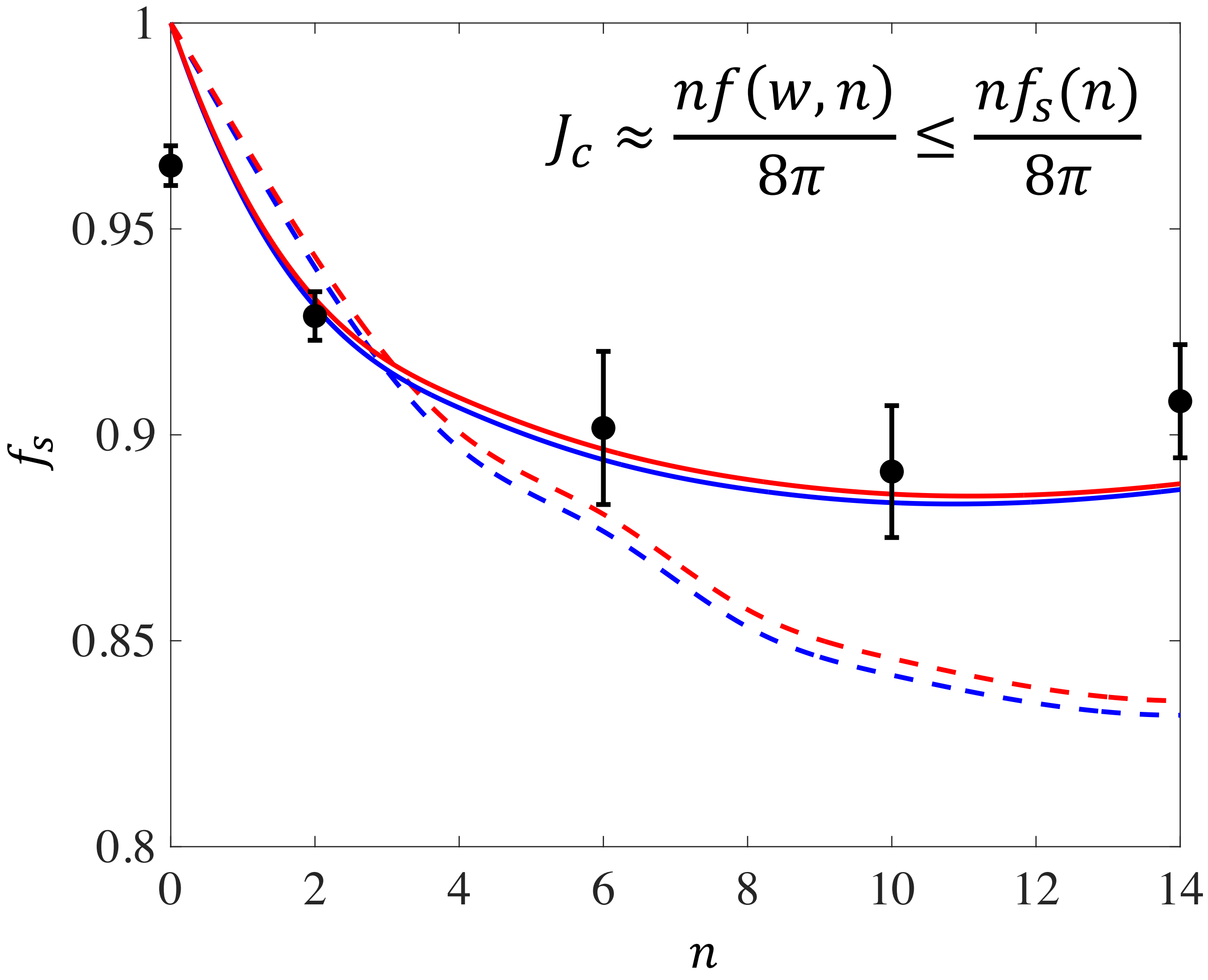
Leggett's Superfluid fraction



$$f_s = 1 - \lim_{\Omega \to 0} \frac{\langle L \rangle}{I_{\mathrm{cl}} \Omega}$$

Leggett PRL 1970

Leggett J. STAT. PHYS. 1998

upper bound to f_s

$$\frac{1}{(2\pi)^2} \int d\theta \, \frac{1}{\iint dz dr \, r \, \rho(r,\theta,z)}$$

lower bound to f_s

$$\iint \frac{dz dr \, r}{\frac{1}{(2\pi)^2} \int d\theta \, \frac{1}{\rho(r,\theta,z)}}$$

bounds including imaging resolution

experimental upper bound from in-situ images

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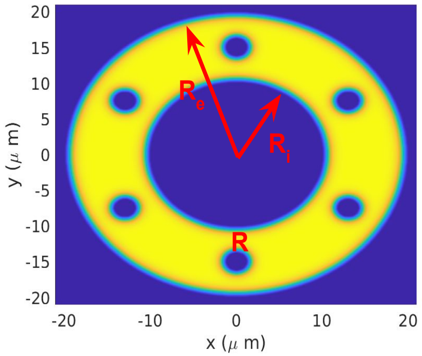
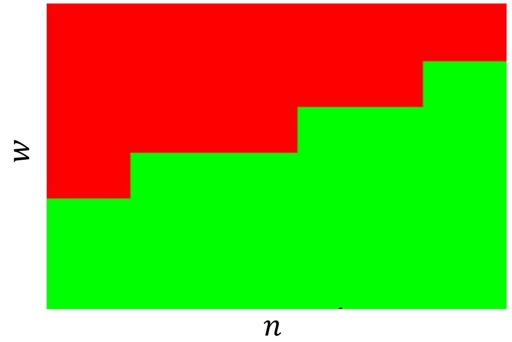
Conclusions and Perspectives

- On demand excitation of persistent currents in superfluid rings and decay via vortex emission
- Generalization to superfluid rings with localized impurities (in ordered or disordered configurations): *preliminary results confirm the stabilization mechanism* (unpublished)





- Fermions: the increasing number of Josephson barriers might compete with dissipative effects, such as Cooper pairbreaking
- Supersolids: density modulations can be associated to effective Josephson junctions

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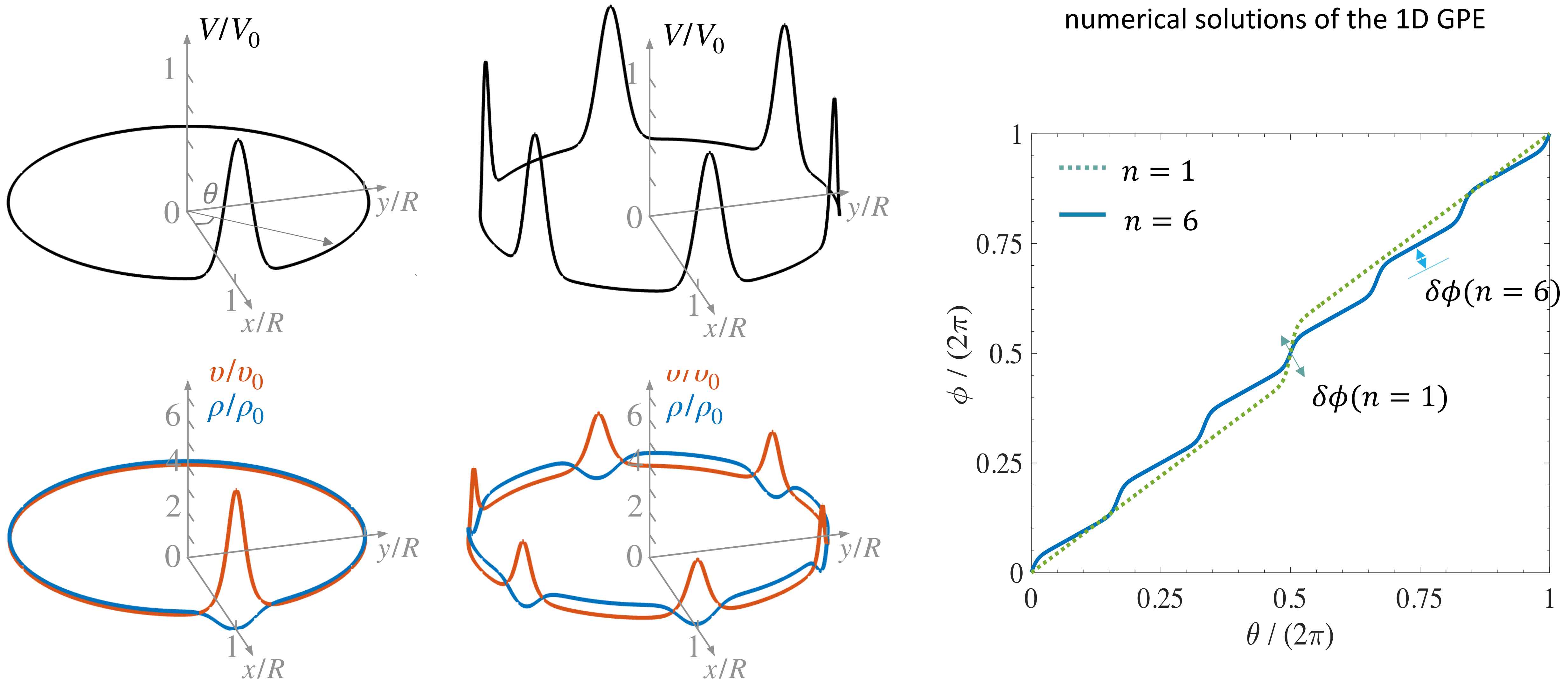
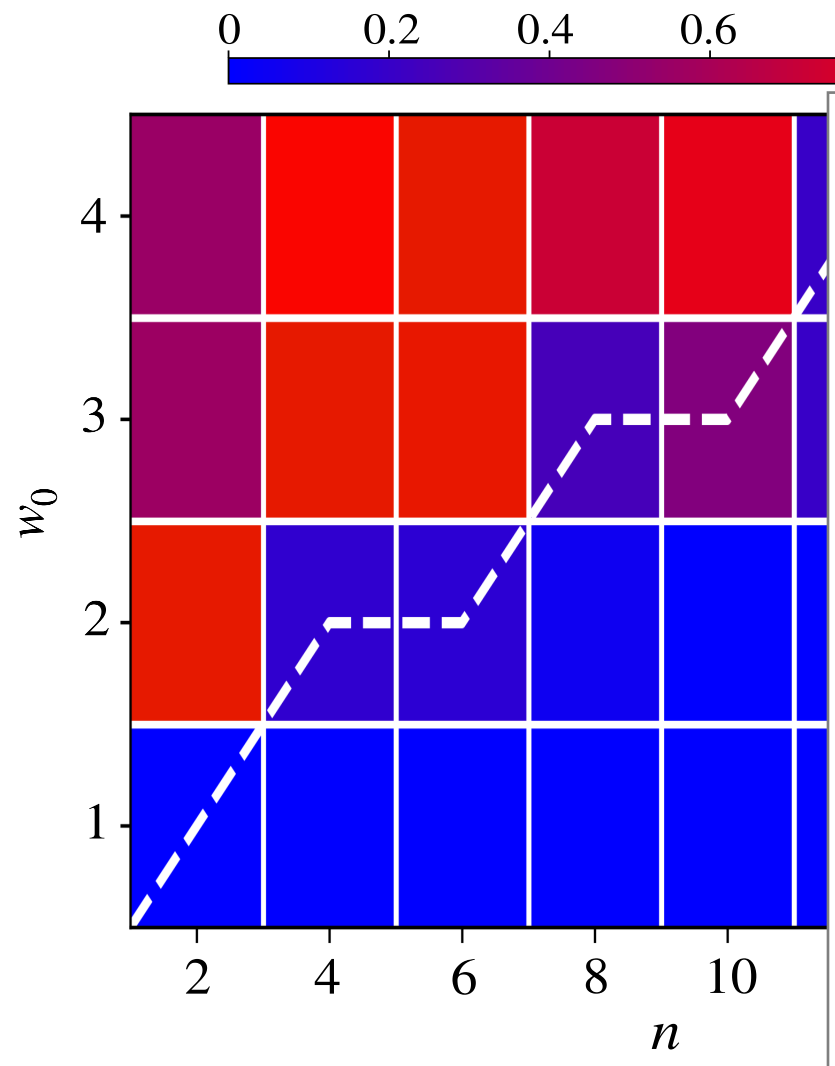
- The extraordinary experimental control paves the way to include quantum fluctuations effects

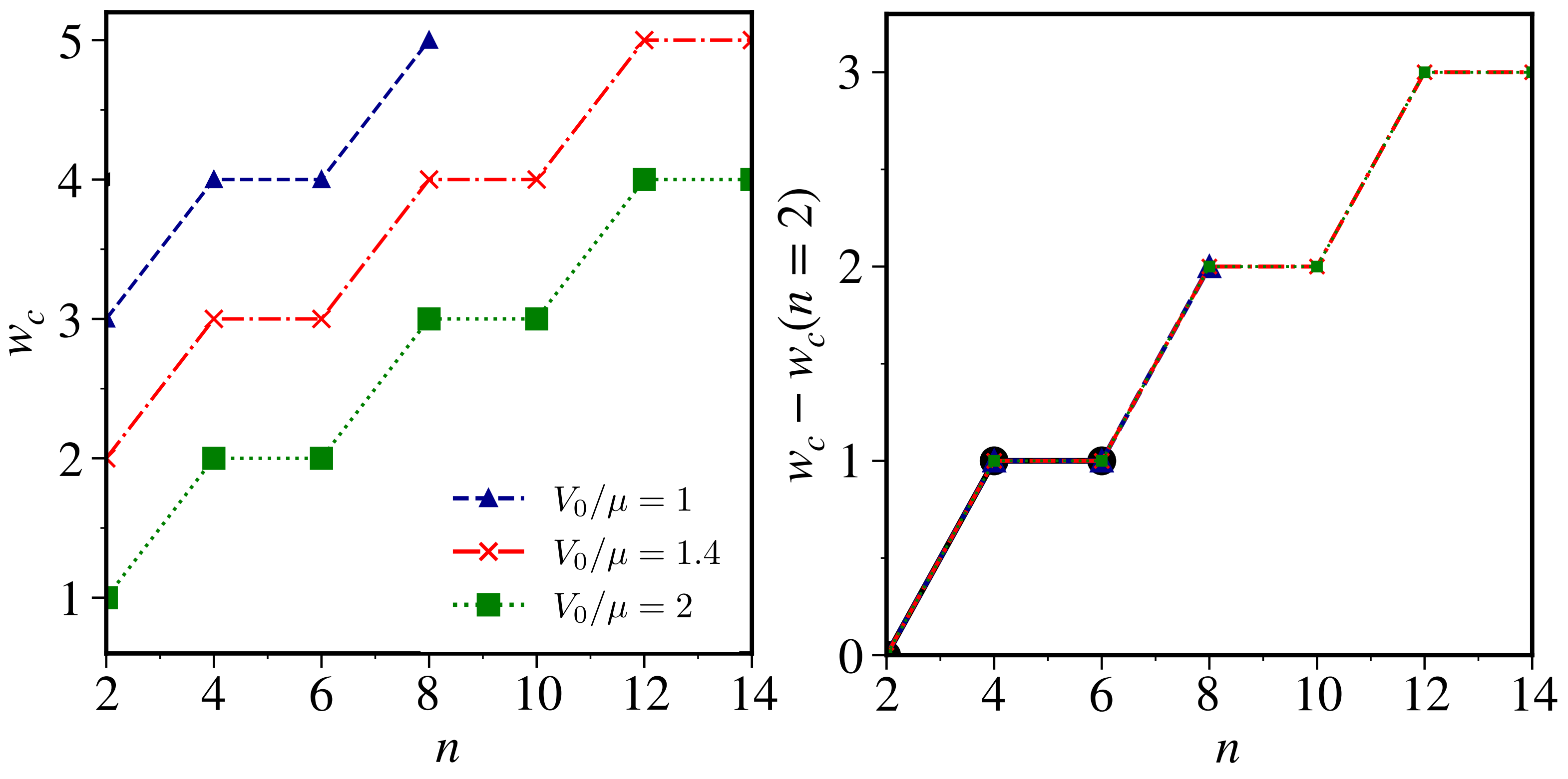
1D beyond the approximation of narrow junctions



Average circulation and stability



dashed line: the critical theoretical line for $V_0/\mu = 1.4$



critical line moves vertically when changing V_0/μ

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