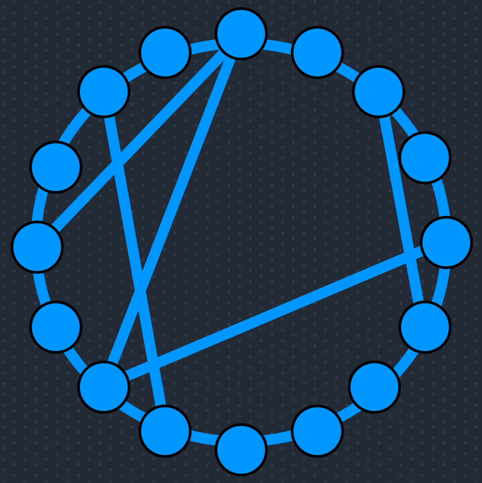
# Emulating long-range interactions via disorder

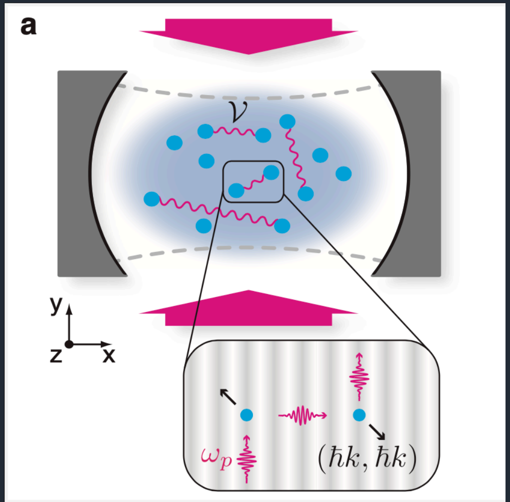
The history of a fight between mathematics and physics

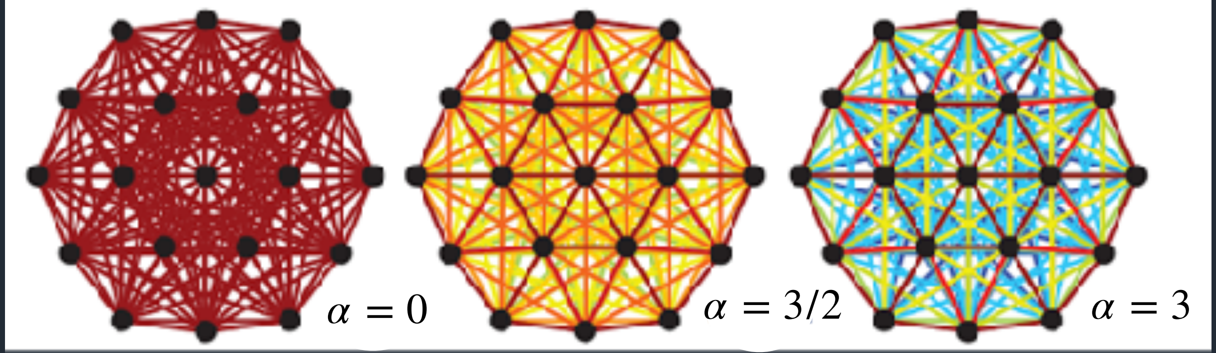


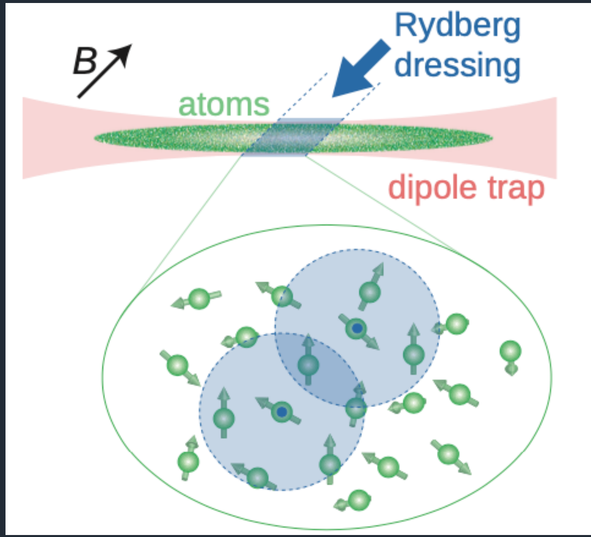
Nicolò Defenu

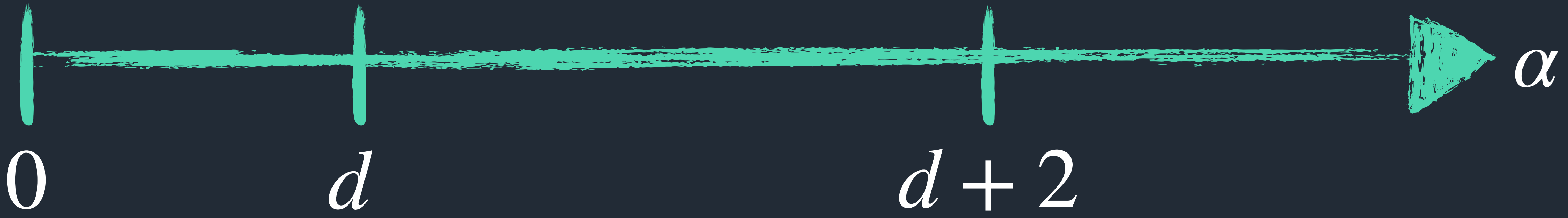
# Long-range quantum simulators

$$V(r) \sim r^{-\alpha}$$



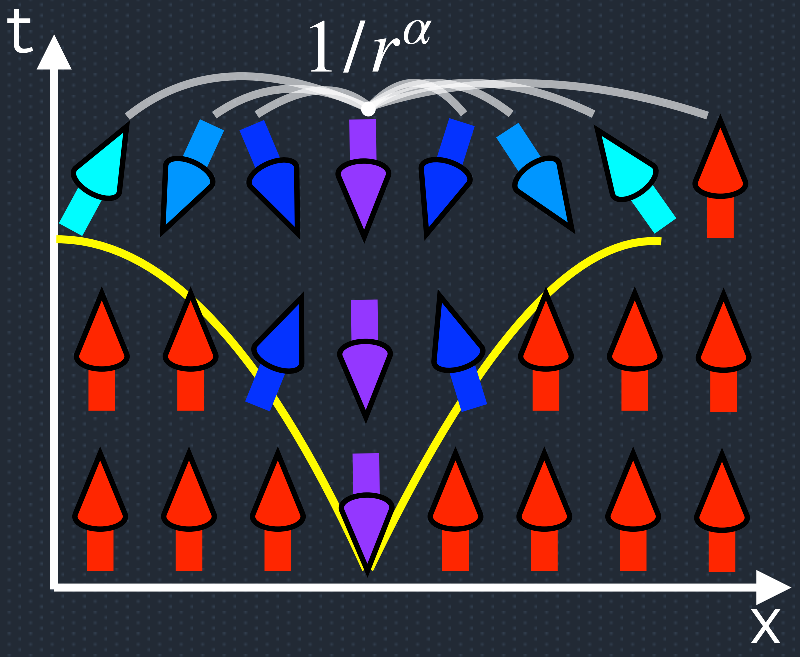




"Long-range interacting quantum systems", ND et al. Rev. Mod. Phys. 95, 035002 (2023).

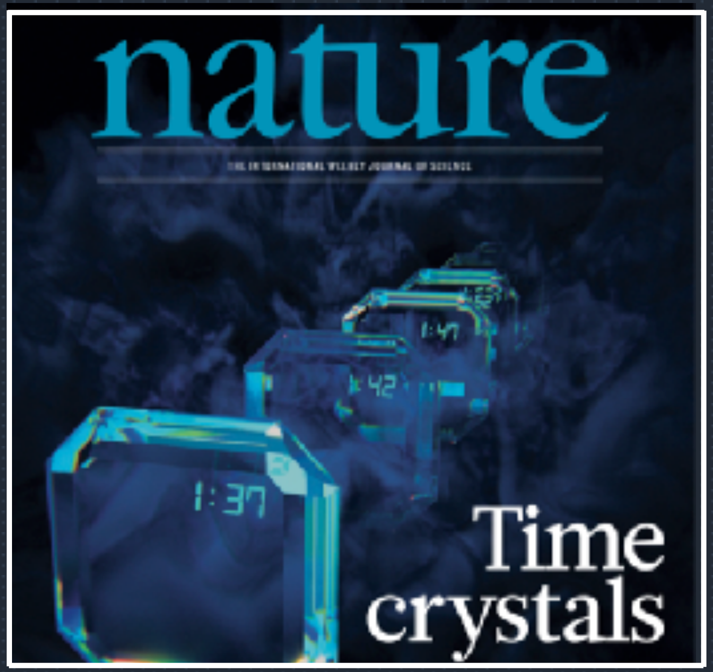
# Novel Dynamical Phenomena



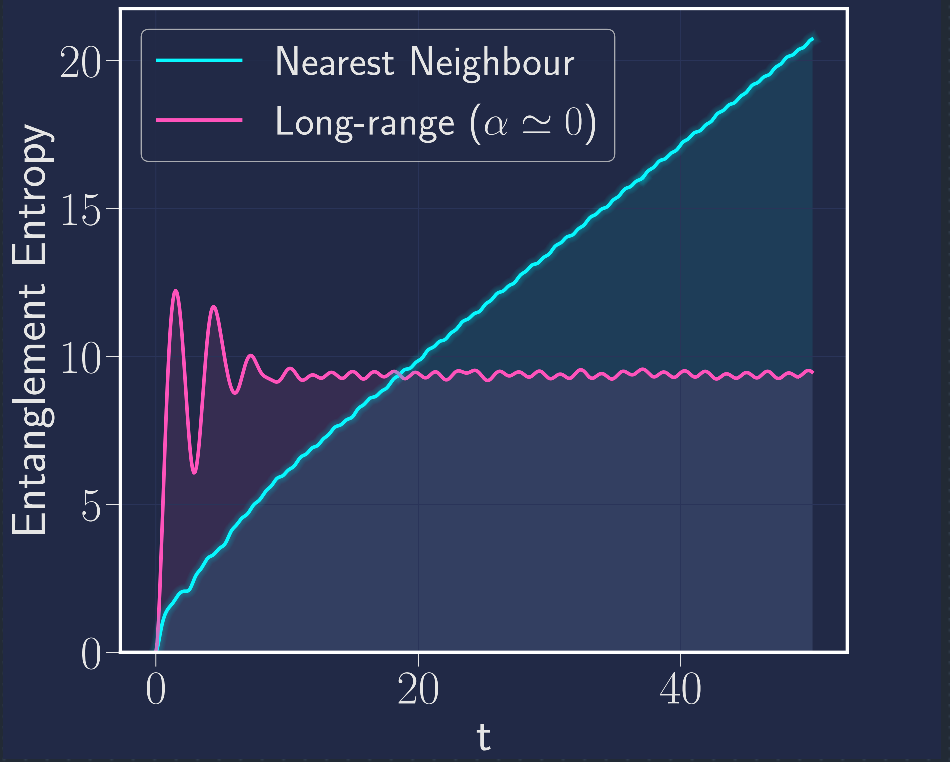
### Fast Signal Spreading




### Time Crystals




### Ergodicity Breaking




P. Richerme, et al., Nature 511, 198 (2014).

J. Zhang, et al., Nature 543, 164 (2017).

# Long-range vs Short-Range effective theories

$$H_{\mathrm{LR}} = \int d^d x \left\{ \partial_\mu^{\sigma/2}\psi \partial_\mu^{\sigma/2}\psi + m\psi^2 + g\psi^4 \right\}$$

**VS**

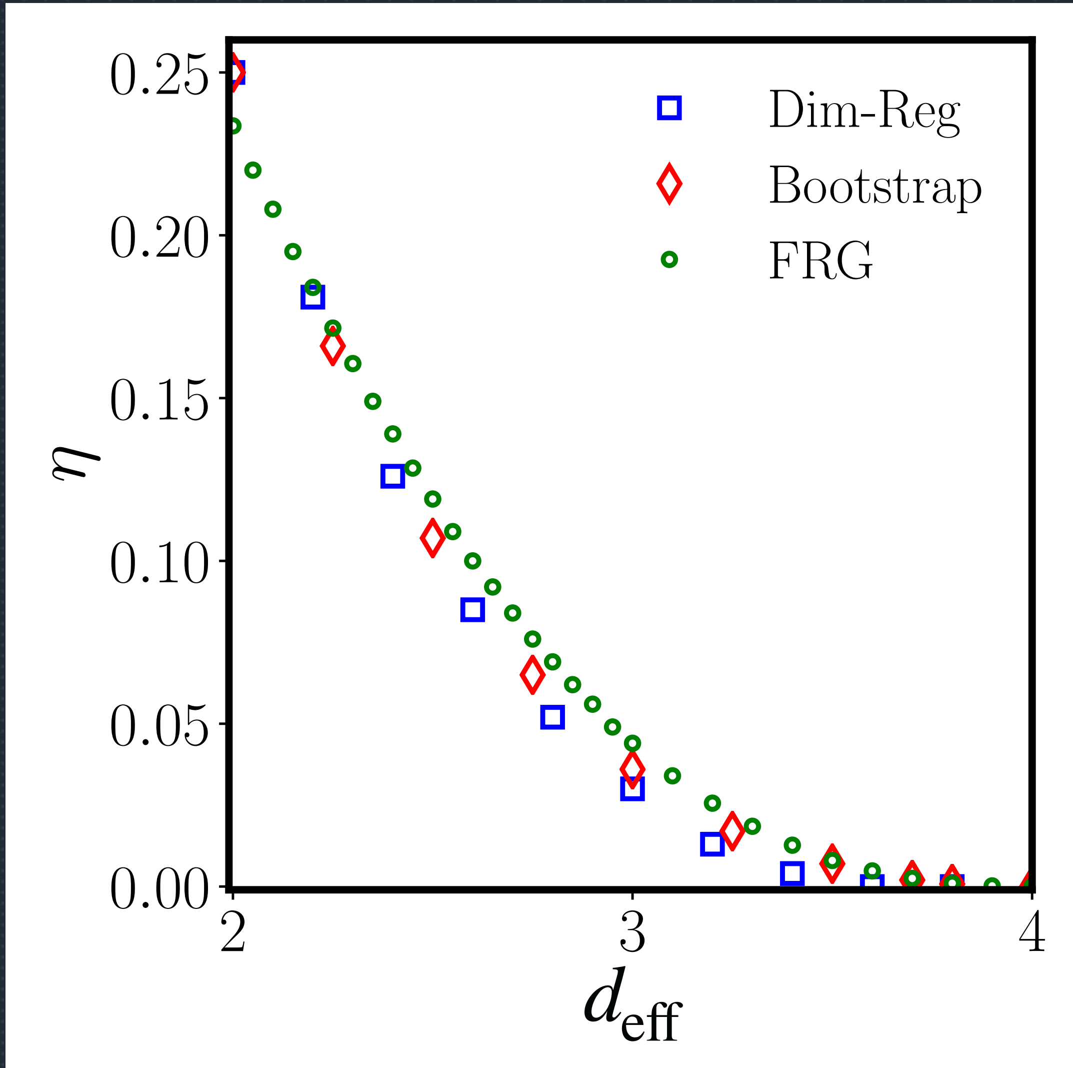
$$H_{\mathrm{SR}} = \int d^{d_{\mathrm{eff}}} x \left\{ \partial_\mu\psi \partial_\mu\psi + m\psi^2 + g\psi^4 \right\}$$

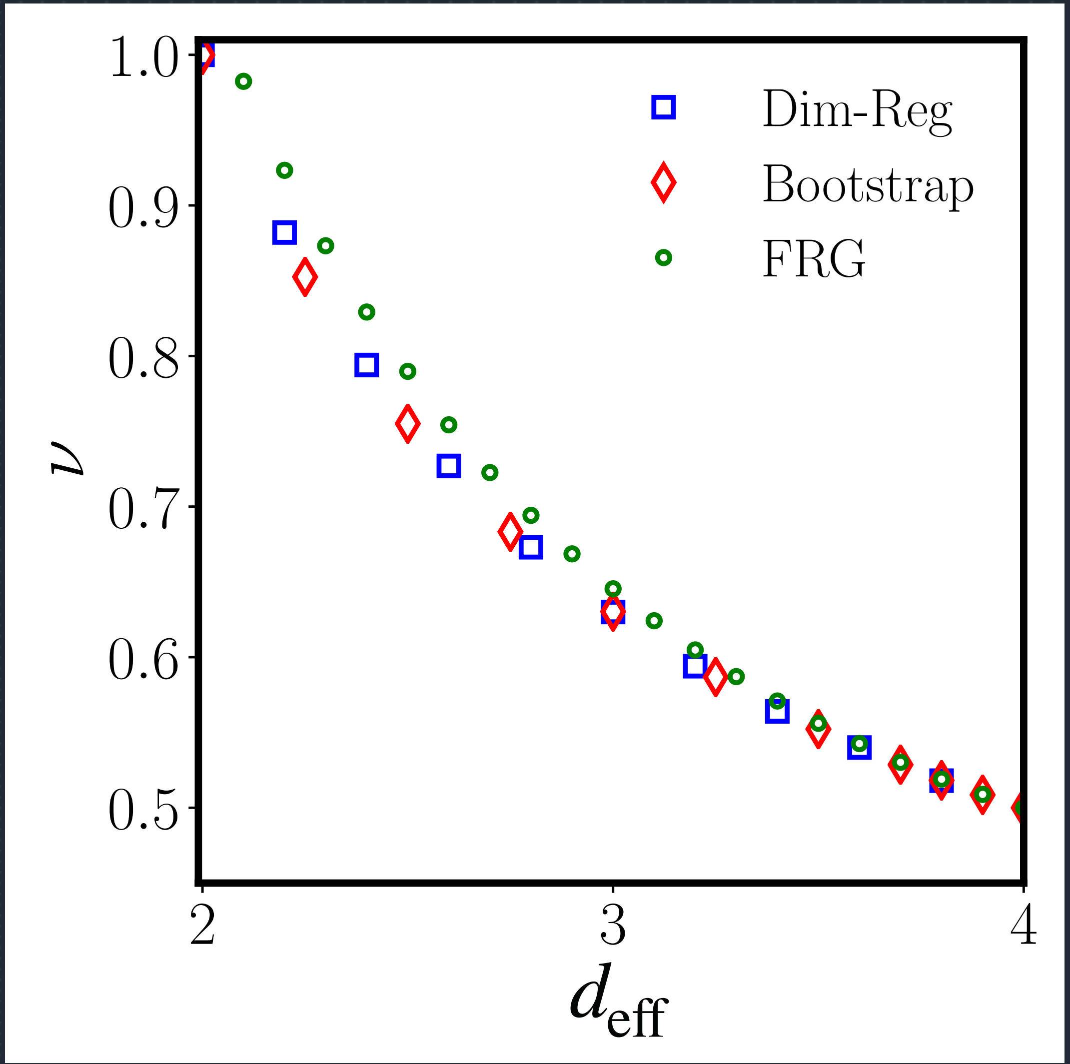
# $\varphi^4$ Theories in fractional dimension



$$\langle\psi(0)\psi(r)\rangle \propto r^{d-2+\eta}$$

$$\xi \propto \varepsilon^{-\nu}$$



# Singular part of the free energies

$$f_{\mathrm{LR}} \approx \frac{1}{V}\Phi_{\mathrm{LR}}(V^{y_\tau^{\mathrm{LR}}/d}\tau, V^{y_h^{\mathrm{LR}}/d}h, V^{y_u^{\mathrm{LR}}/d}u)$$

**Can they be equal?**

$$f_{\mathrm{SR}} \approx \frac{1}{V}\Phi_{\mathrm{SR}}(V^{y_\tau^{\mathrm{SR}}/d_{\mathrm{eff}}}\tau, L^{y_h^{\mathrm{SR}}/d_{\mathrm{eff}}}h, V^{y_u^{\mathrm{SR}}/d_{\mathrm{eff}}}u)$$

# Effective dimension relation



$$d\nu_{\mathrm{LR}} = d_{\mathrm{eff}}\nu_{\mathrm{SR}}, \qquad \frac{2-\eta_{\mathrm{LR}}}{d} = \frac{2-\eta_{\mathrm{SR}}}{d_{\mathrm{eff}}}$$

$$\gamma_{\mathrm{LR}} = \gamma_{\mathrm{SR}}, \qquad \omega_{\mathrm{LR}}/d = \omega_{\mathrm{SR}}/d_{\mathrm{eff}}$$

1) One Loop $O(\varepsilon)$-expansion ✓

2) First order $1/N$-expansion ✓

3) Local Potential Approximation (FRG) ✓

# Just an approximation



**Functional RG study beyond LPA:** ***Approximate but accurate within 5%***

N. Defenu, A. Trombettoni, A. Codello, *Phys. Rev. E 92, 052113 (2015).*

**Physics perspective VS Mathematical result**

***"[…] This 'effective dimension' is clearly not a fundamental notion […]."***

C Behan, et al, J. Phys. A: Math. Th. 50 (35), 354002 (2017).

# Effective dimension is not exact

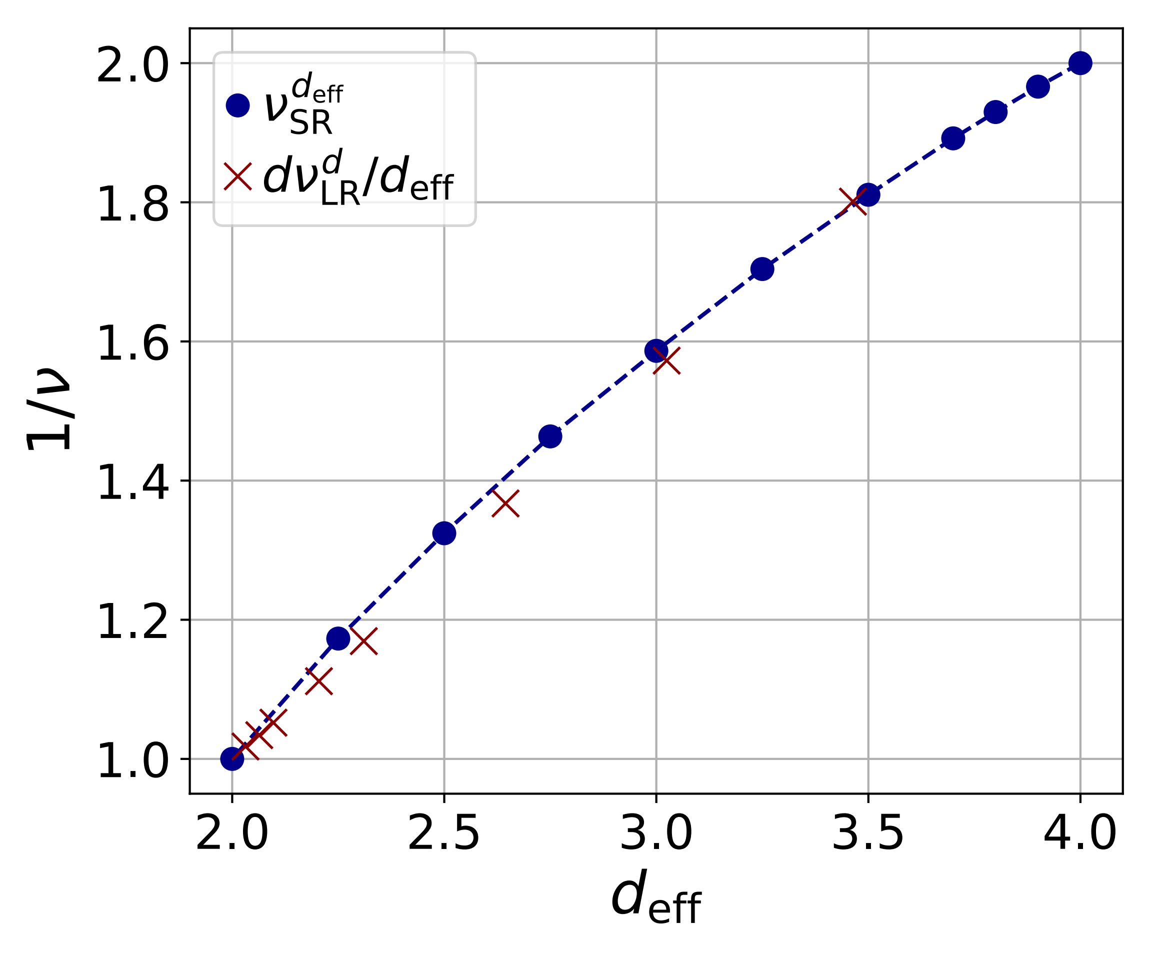


**Nearest neighbour model (CFT Data)** O

Sheer El-Showk, et al. Phys. Rev. Lett. 112, 141601 (2014)

**Long-range model (CFT Data)** ✗

Connor Behan, et al. JHEP2024 136 (2024)




A. Solfalli & N. Defenu Phys. Rev. E 110, 044121 (2024)

# The Spectral Dimension



### Vibrational spectrum

At low frequencies the vibrational spectrum of coupled oscillators on a network obeys:

$$\rho(\omega) \propto \omega^{d_s - 1}$$

### Random Walk Return Rates

A random walker hopping on networks will return at its starting point with probability:

$$P_0(t) \propto t^{-d_s/2}$$

### Mermin-Wagner Theorem

Spontaneous symmetry breaking of continuous symmetries is forbidden for

$$d_s \leq 2$$

## Does it control universal properties?

*What about discrete symmetries?*

1)R. Burioni, D. Cassi, *Phys. Rev. Lett.* ***76***, *1091 (1996)*.

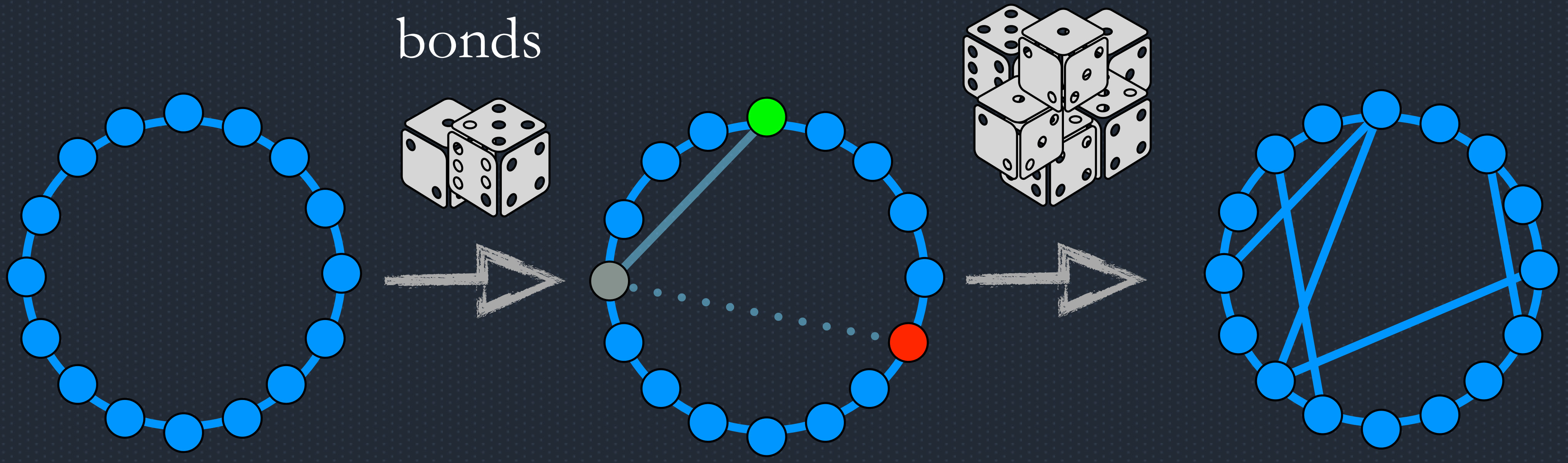
2)R. Burioni, D. Cassi, A. Vezzani, *Phys. Rev.* ***E 60***, *1500 (1999)*.

# A local model in fractional dimension?



Additional random bonds

$\sim N$ new bonds



Regular lattice in $d$-dimension

$$p \sim \frac{1}{r_{ij}^{\alpha}}$$

Self-averaging graph structure

A. P. Millán, G. Gori, F. Battiston, T. Enss, N. Defenu, *Phys. Rev. Res.* 3, 023015 (2021).

# Complex networks with tuneable dimension

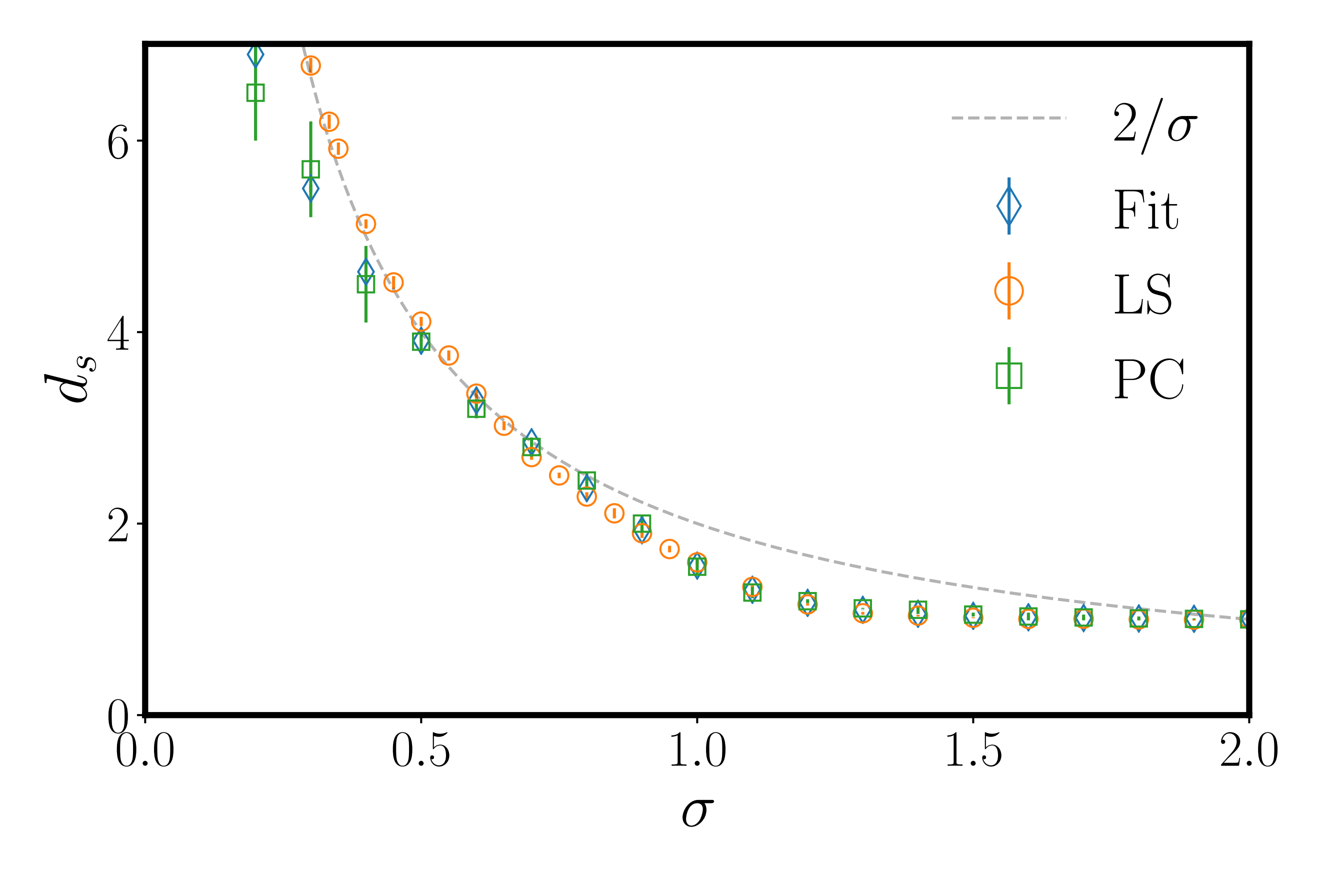


### Collapse of the return probabilities

The average of the random walker return rates for several network realisations with different number of sites are collapsed to obtain an estimation of the spectral dimension.

### Scaling of the smallest eigenvalues

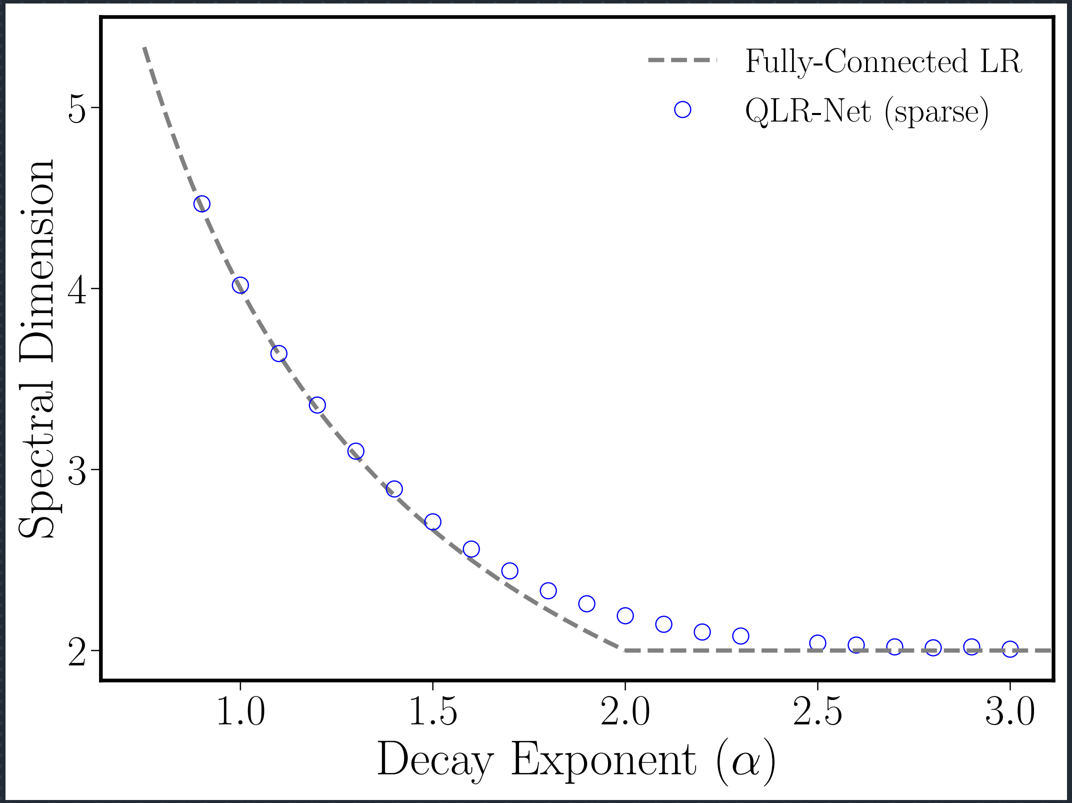
The power law scaling of the first eigenvalue of the laplacian operator as a function of the network size is studied for large network sizes.



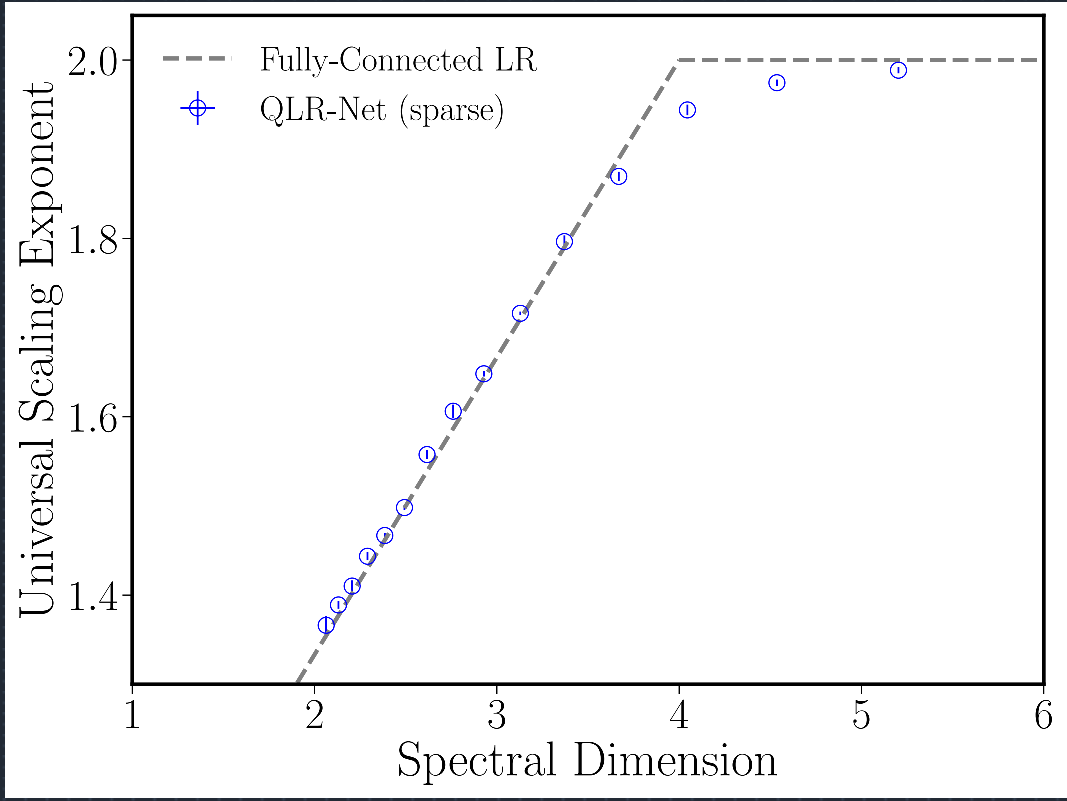

Research funded by the STRUCTURES collaboration under exploratory project (EP) 2.3: “Universality on Network Structures from Quantum Dynamics to Big Data”. PI: ND

# The universality conjecture

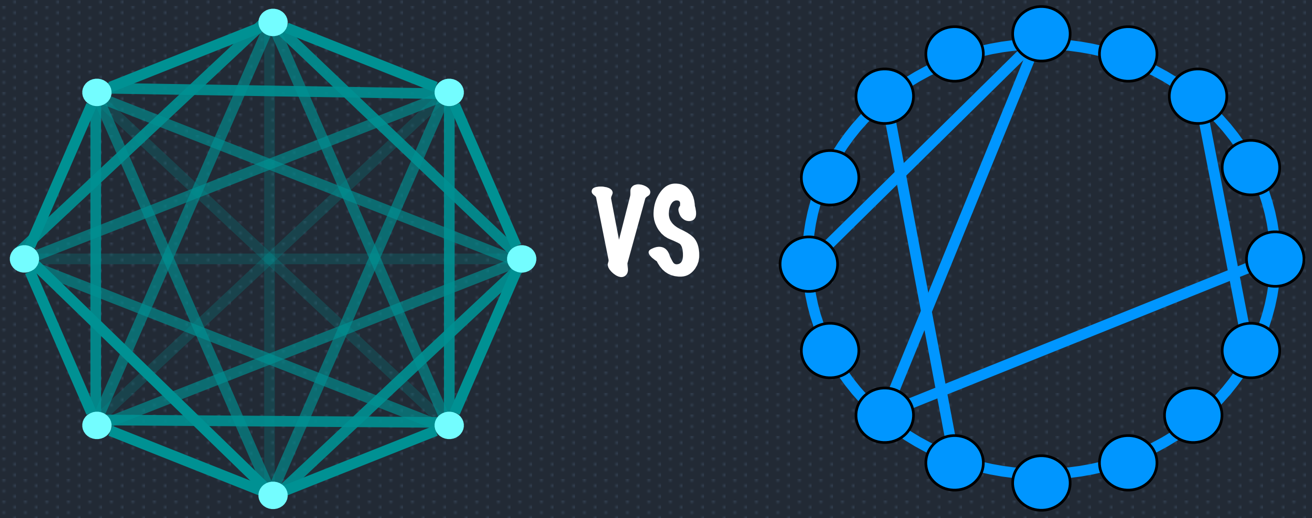
## Match the low-energy spectra




## Reproduce the universal scaling



# My goal




Fully connected

Sparse

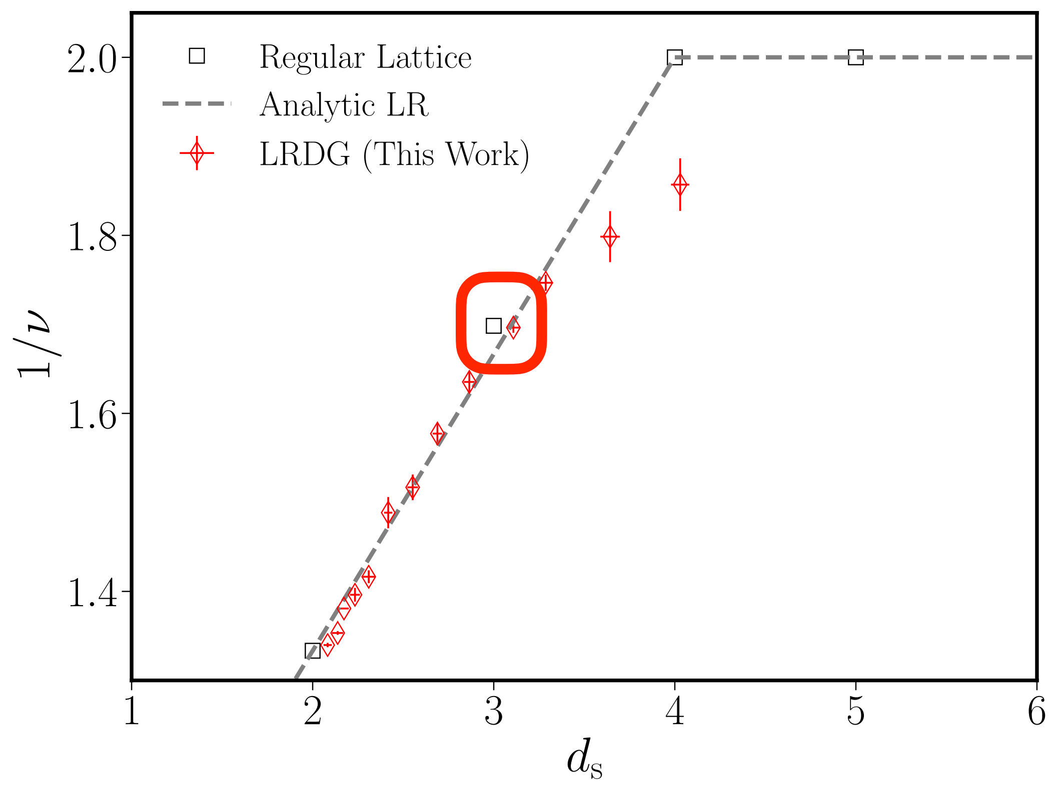
Advantages:

- Fast entanglement spreading inherited by long-range couplings
- Amenable numerics thanks to sparse coupling matrices
- Universal tool to describe long-range interacting systems

# Open Questions



## Reproduce the universal scaling




### MC Simulations on Regular Lattice

Slade Gordon, 2019, Self-avoiding walk, spin systems and renormalization Proc. R. Soc. A.47520180549 20180549

### MC Simulations on LR diluted graph

The critical exponent is extracted by the finite size scaling of the gyration ratio of the walk length.

G. Bighin, T. Enss, N. Defenu, Nat. Comm. 15, 4207 (2024).

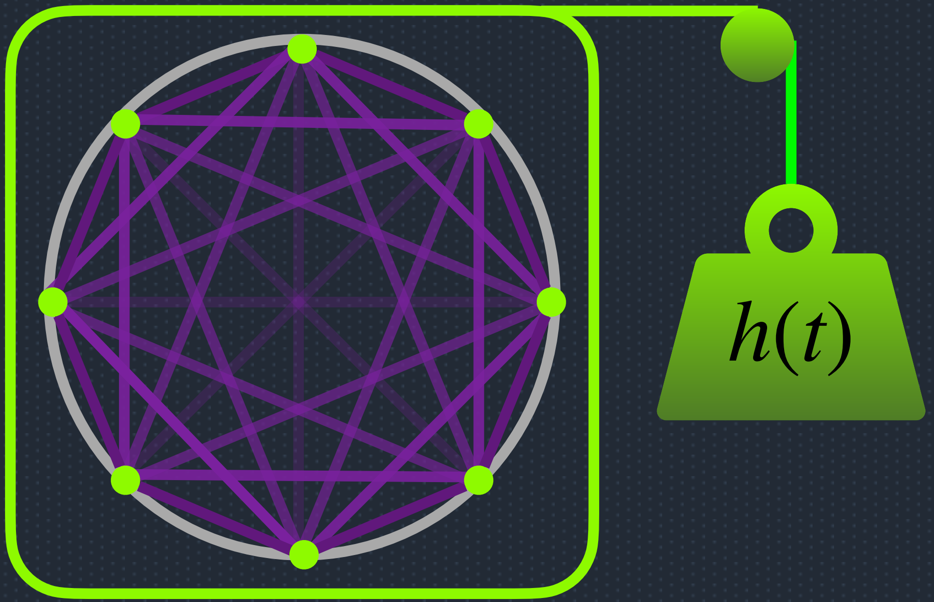
# Entering a new era of quantum information



Manipulate fundamental physics laws via complex geometric structures

Tuneable Spectrum + Strong Interactions

Novel fundamental physics phenomena




Novel proposals for Quantum Technologies

# Collaborators and funding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Ana P. Millan


Stefano Ruffo


Andrea Trombettoni


Giacomo Gori


Federico Battiston

erc
European Research Council
Established by the European Commission

Starting Grant: Quantum Long-Range Networks

Swiss National Science Foundation

Project Funding [200021_207537]

# Thank You