

Quantum Turing Pattern Selection in a Dissipative Bosonic Chain

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Abstract: We present a dissipative bosonic chain where engineered nonlocal Lindblad channels induce quantum Turing-like mechanisms. Using a Wigner phase-space approach, we derive an effective semiclassical Fokker–Planck description enabling systematic analysis of instabilities and mode selection.

Spatial pattern formation is a feature of classical reaction-diffusion systems, whereas quantum realizations of Turing-type behavior have so far relied mainly on external measurement-feedback loops or few-mode symmetry-breaking scenarios [1,2]. Here we show that discrete Turing mode selection can instead arise intrinsically in a fully microscopic open quantum chain. We consider a one-dimensional chain of $N \geq 2$ bosonic modes governed by a GKSL master equation with local degenerate optical parametric oscillator driving, single-photon loss, nonlinear two-photon saturation, a nearest-neighbor gradient pump, and a curvature-damping channel. The full dynamics is described by the Quantum Master Equation (QME):

$$\dot{\hat{\rho}} = -i[\hat{H}_{loc}, \hat{\rho}] + \sum_{j=1}^N (\gamma_1 \mathcal{D}[\hat{a}_j] \hat{\rho} + \gamma_2 \mathcal{D}[\hat{a}_j^2] \hat{\rho}) + \sum_{j=1}^{N-1} \mathcal{D}[\hat{L}_j^{(\lambda)}] \hat{\rho} + \sum_{j=1}^N \mathcal{D}[\hat{L}_j^{(\kappa)}] \hat{\rho}$$

At the mean-field level, the nonlocal dissipative channels produce two spatial operators that compete with each other: a short-range anti-diffusive channel and a long-range diffusive one. The combination of both effects results in an anti-diffusive discretized Laplacian that is stabilized by a bi-Laplacian under no-flux boundary conditions, providing analytical instability thresholds and discrete modes selection. By evaluating the QME on the semiclassical amplitudes $\alpha_j = \langle \hat{a}_j \rangle$, one obtains the mean-field equations:

$$\dot{\alpha}_j = \left(\gamma_2 - \frac{\gamma_1}{2} - i\Delta \right) \alpha_j - \gamma_2 |\alpha_j|^2 \alpha_j - 2\eta e^{i\theta} \alpha_j^* + \frac{\lambda}{2} \sum_k (\nabla^{(2)})_{jk} \alpha_k - \frac{\kappa}{2} \sum_k (\nabla^{(4)})_{jk} \alpha_k$$

The most direct quantum signature of the patterned regime appears in the reduced Wigner functions. For the three-site chain, under the choice of the parameters that allow for the Turing instability, the reduced W_1 and W_3 become bimodal, with maxima close to $\alpha_1 = -\alpha_3 \approx \pm(1, -1)$, while W_2 remains unimodal around $\alpha_2 = 0$, clear sign of spatial **non-homogeneity** between the three sites (Turing-like mechanism).

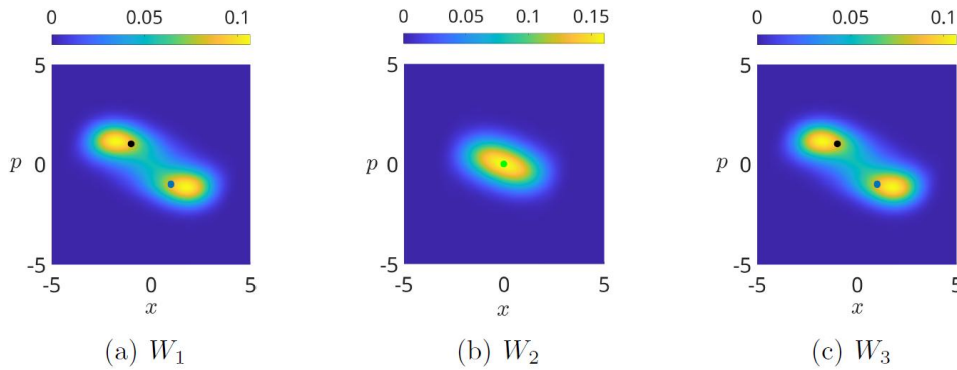


Fig. 1 Reduced Wigner functions W_1, W_2 and W_3 . The bimodal structure on sites 1 and 3, together with the unimodal distribution on site 2, reveals **non-homogeneity** between the three sites (Turing-like mechanism).

While the quantum steady state preserves the underlying symmetry as a statistical mixture of symmetry-related classical patterns, by tuning the system parameters, we can access a broad spectrum of spatial non-homogeneities, leading also to the emergence of quantum version of the Turing wave instabilities. This highlights a controllable route to complex patterning in multimode bosonic chain.

References

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