Quantum Fisher information as a witness of non-Markovianity and criticality in the spin-boson model

Daniele Parlato[1], **Grazia Di Bello**[1,3], **Fabrizio Pavan**[1] , **Carmine Antonio Perroni**[2,3], **Giulio De Filippis**[2,3*]

1. Dip. di Fisica E. Pancini - Università di Napoli Federico II - I-80126 Napoli, Italy

2. SPIN-CNR and Dip. di Fisica E. Pancini - Università di Napoli Federico II - I-80126 Napoli, Italy

3. INFN, Sezione di Napoli - Complesso Universitario di Monte S. Angelo - I-80126 Napoli, Italy

** Member of NQSTI*

Abstract: We analyze the static and dynamic dissipative behavior of quantum Fisher information for a qubit coupled to a bosonic bath, revealing divergences near the quantum phase transition and at vanishing coupling, and non-Markovian dynamics.

We study the Quantum Fisher Information matrix [1,2] (QFIM) in the spin-boson model with added dephasing noise, a paradigmatic model for studying a qubit's dissipative behavior. To do so, we use exact numerical methods that represent open quantum systems states as matrix product states (MPSs). The system undergoes a quantum phase transition when varying the qubit-bath coupling α [3]; near the transition, the quantum state becomes maximally sensitive to variations of the coupling, which is reflected into maximal QFI. Here we focus on the coupling-coupling QFIM element $F_{\alpha\alpha}$. We compute the static QFI $F_{\alpha\alpha}$ with no dephasing $\kappa = 0$ and various external magnetic fields h (see Fig. 1 (a)). It shows sharp peaks at the critical points, confirming its role as a marker of criticality. For vanishing coupling $\alpha \to 0$, it exhibits divergent non-perturbative behavior $F_{\alpha\alpha} \sim 1/\alpha$, which we characterize through analytic calculations. The QFI also proves to be a robust marker in the presence of an added pure dephasing noise channel since it still peaks near criticality (see Fig. 1 (b)). We study the dynamics $F_{\alpha\alpha}(t)$ in the coherent regime (Fig. 1 (c)), comparing them to Lindblad's Markovian approximation. The exact curve shows oscillatory behavior, considered a witness of non-Markovianity due to information backflow [4], and converges towards its ground state value. The Lindblad curve does not show these oscillations, further proving that these cannot be replicated within the Markovian framework. These findings strengthen the connection between the QFI and phase transitions, as we observe clear peaks at the known critical point and in the presence of pure dephasing noise. Moreover, we characterize the divergence and show that the dynamical behavior reveals non-Markovianity.

In ongoing work, we are extending this analysis to a regime where pure dephasing is dominant and amplitude damping is minimal, opposite to the parameter setting considered in this article. Preliminary results indicate the existence of two distinct sensing regimes, whose origin remains under further investigation.

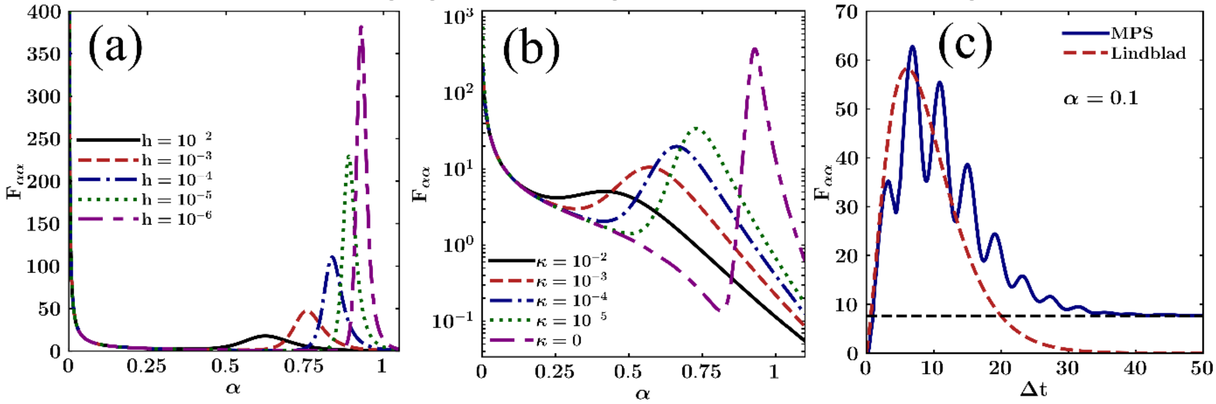

Fig. 1 Static Quantum Fisher Information [1] with $\kappa = 0$ (a) and $\kappa \neq 0$ (b) and dynamic QFI $F_{\alpha\alpha}(t)$ for $\kappa = 0$ and $h = 10^{-6}$ (c)

References

[1] D. Parlato, G. Di Bello, F. Pavan, G. De Filippis, C. A .Perroni, "Quantum Fisher information as a witness of non-Markovianity and criticality in the spin-boson model", Phys. Rev. B **112,** 224314 (2025)

[2] J. Liu, H. Yuan, X.-M. Lu, and X. Wang, "Quantum Fisher information matrix and multiparameter estimation", J. Phys. A: Math. Theor. **53**, 023001 (2019)

[3] G. De Filippis, A. de Candia, L. M. Cangemi, M. Sassetti, R. Fazio, and V. Cataudella, "Quantum phase transitionsin the spin-boson model: Monte Carlo method versus variational approach à la Feynman", Phys. Rev. B **101**, 180408 (2020).

[4] X.-M. Lu, X. Wang, and C. P. Sun, "Quantum Fisher information flow and non-Markovian processes of open systems", Phys. Rev. A **82,** 042103 (2010).