Non-Markovian Dynamics of Two Qubits Coupled via a Finite Transmission Line

Fabio Borrelli[1], **Giovanni Miano**[1,2], **Carlo Forestiere**[1]

1. *Università degli Studi di Napoli Federico II, Napoli, 80125, Italy*
2. *Scuola Superiore Meridionale, Napoli, 80125, Italy*

Abstract: We study two superconducting qubits coupled through a finite transmission-line, derive a Hamiltonian in which the line modes separate into even- and odd-sectors, and use HEOM and BLP-measure to show non-Markovianity of the two-qubit dynamics.

We study the reduced dynamics of two superconducting qubits capacitively coupled to a finite transmission line. Starting from circuit quantization [1-2], we derive the circuit Hamiltonian in which the line modes separate into even- and odd-parity sectors, that couple to the qubits through collective operators. We demonstrate that in the long-line limit, each sector admits an analytic continuum description in terms of a physically parametrized Drude--Lorentz spectral density, allowing us to connect a circuit model to an open quantum system description. To capture dynamical features beyond the reach of Markovian approximations, we employ the Hierarchical Equations of Motion method [3]. This non-perturbative framework allows us to investigate the impact of the environmental spectral structure on the reduced dynamics of the qubits. By combining dynamical observables with the Breuer--Laine--Piilo measure of non-Markovianity [4], we identify the parameter regimes in which a Markovian description remains reliable and those in which memory effects play a central role.

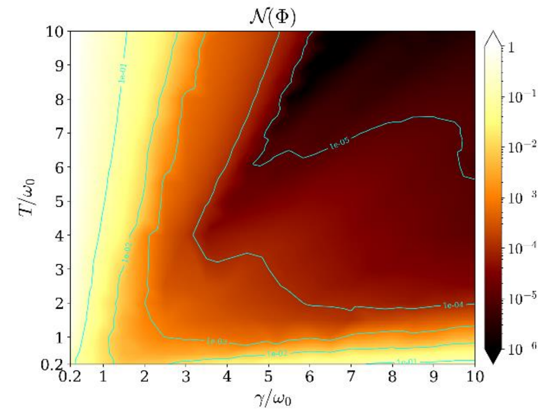

Fig. 1 BLP non-Markovianity measure N(Φ), characterizing the dynamics of two qubits coupled through a finite transmission line, as a function of the bath temperature T/ω_0 (vertical axis) and cutoff frequency γ/ω_0 (horizontal axis), with Ω and T up to $10\omega_0$. The renormalization energy is fixed at $\lambda = 0.1\omega_0$.

In particular, we observe non-exponential relaxation, information backflow, and memory-assisted collective phenomena arising from the structured environment. Our results show that, for typical temperatures and cutoff frequencies relevant to superconducting quantum circuits, the dynamics lies in a non-Markovian regime. These findings provide a quantitative characterization of memory effects, clarifying when non-Markovian methods are required to accurately describe the dynamics of coupled superconducting qubits.

Example References

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