Ergotropy and Dynamical Signatures of Many-Body Localization and Discrete Time Crystals in Disordered Heisenberg Chains

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Abstract: We show that local ergotropy, the extractable work via local transformations, is a marker for ergodic-to-localized phases transition and we demonstrate the emergence of Discrete Time Crystals phase under periodic driving of Heisenberg localized chains.

Many-Body Localization (MBL) and **Anderson Localization (AL)** represent a fundamental paradigm of non-ergodic quantum dynamics in disordered systems. MBL and AL are hallmarks of interacting and non-interacting systems, respectively. From a quantum-information perspective, MBL phase is uniquely identified by the logarithmic entanglement growth, while in the AL case the entanglement is stationary over time. We investigate anisotropic disordered Heisenberg spin chains through state-of-the-art numerical simulations [1].

We propose **local ergotropy** - defined as the maximum work extractable via local unitary operations on a small subsystem - as a robust thermodynamic witness of the transition from the ergodic phase to MBL and AL phases. We demonstrate that within the MBL phase, both local ergotropy, reported in Fig.1, and its quantum fluctuations exhibit a slow, logarithmic temporal evolution, mirroring the phenomenology of entanglement entropy. Our findings suggest that leveraging local control provides a novel indicator of localization based on extractable work, offering a thermodynamic alternative to standard entropic measures [1].

Furthermore, we explore the emergence of **discrete time crystals (DTC)** when an external drive is applied to such localized systems. A DTC is an out-of-equilibrium phase of matter in which continuous time-translation symmetry is spontaneously broken. As a consequence, the system exhibits a subharmonic response, and observables become periodic functions with a period that is an integer multiple of that of the drive.

While DTC phases have been extensively studied in Ising-like models, we unveil their robustness in disordered Heisenberg chains [2]. By analyzing the dynamics of ergotropy, entanglement, and spin-spin spatial correlations, we identify clear signatures that characterize the transition from the DTC phase to the MBL regime as the driving parameters are tuned. Our results provide new insights into the stability of out-of-equilibrium phases in many-body quantum systems.

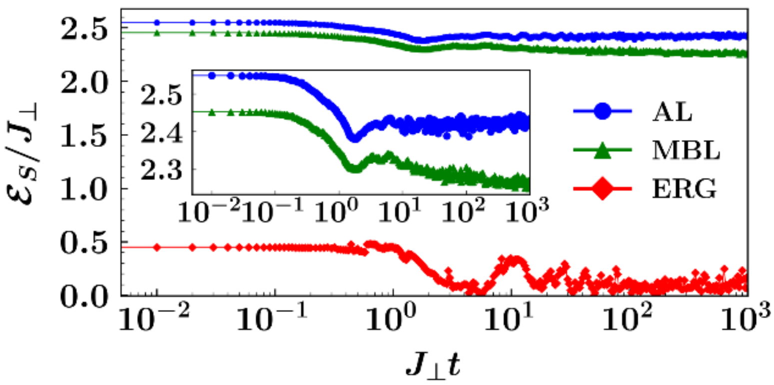

Fig. 1 Local ergotropy as a function of time in the three phases: AL (blue circle), MBL (green triangle) and ERG (red diamond). The inset provides a zoomed-in view of the local ergotropy as a function of time for the two localized phases. Results are obtained by averaging over 10^3 disorder realizations for a chain length N=8.

Example References

[1] **F. Formicola, G. Di Bello, G. De Filippis, V. Cataudella, D. Farina, and C. A. Perroni**, "Local ergotropy, dynamically witnesses many-body localized phases," Phys. Rev. Res., **vol. 7**, p. 043 086, (2025), doi: 10.1103/2z1g-rgr9.

[2] **F. Formicola**, et al. , to be submitted.