# Convex optimization for open quantum systems and certification tasks

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**Abstract**: We devise novel methods that exploit convex optimization in quantum technology: from certifying properties of quantum systems under incomplete information, to solving problems in open systems such as regularizing master equations while preserving non-Markovian effects.

Convex optimization and, in particular, semidefinite programming represent fundamental tools in quantum information science and emerging trends in open quantum many-body physics [1, 2].

We present here novel use cases in two different contexts, exploiting the potential of performing projections on convex sets and obtaining certified bounds for quantities of interest.

In particular, we employ projection techniques in the context of open quantum systems [3] where a relevant problem is the lack of complete positivity of dynamical maps obtained after weak-coupling approximations, a famous example being the Redfield master equation. A number of approaches exist to recover well-defined evolutions under additional Markovian assumptions, but much less is known beyond this regime. We propose a numerical method to cure the complete-positivity violation issue while preserving the non-Markovian features of an arbitrary original dynamical map. The idea is to replace its unphysical Choi operator with its closest physical one (see Fig. 1), mimicking recent work on quantum process tomography (see, e.g., [4]). We also show that the regularized dynamics is more accurate in terms of reproducing the exact one.

Then, we deal with certification of quantum state properties under partial information [5]. Convex and concave functions of quantum states play a key role in quantum physics, with examples ranging from Bell inequalities to von Neumann entropy. However, in experimental scenarios, direct measurements of these functions are often impractical. We address this issue by introducing two methods for determining rigorous confidence bounds for convex functions based on informationally incomplete measurements and finite statistics. Our approach outperforms existing protocols by providing tighter bounds for a fixed confidence level and number of measurements. We evaluate the performance of our methods using both numerical and experimental data. Our findings demonstrate the efficacy of our approach, paving the way for improved quantum state certification in real-world applications.

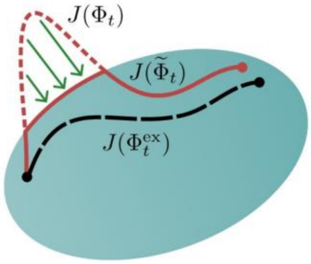


**Fig. 1** Schematic on the Choi-proximity regularization of Ref. [3].

## References

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