Local ergotropy: connection with quantum phase transitions and its “extended” versions

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Abstract: We investigate interplays of local ergotropy with many-body phenomena and we devise extended versions of local ergotropy to overcome formal inconsistencies and obtain practical advantages.

Introduction.— While for closed systems maximum quantum work extraction is given by the *ergotropy*, this question is unclear in systems interacting with an environment. While first approaches relied on weak-coupling assumptions [1], later, at arbitrary coupling, the concept of local ergotropy was proposed [2] as the maximum extractable work from the system-environment compound by applying a local unitary on the system [Fig. 1(a)].

Results.— Being interested in potential connections with many-body phenomena beyond the Markov approximation, in [3] we start by investigating a two-qubit multimode Rabi model [Fig. 1(b)] focusing on local ergotropy within a parameter regime where a Berezinskii-Kosterlitz-Thouless dissipative phase transition occurs [4]. We define a protocol for charging, storing in quasi-decoherence free subspaces, and discharging the two-qubit system, interpreted as the working principle of an open quantum battery. We further examine the impact of the phase transition on local ergotropy and identify potential markers based on it.

From a more fundamental point of view, we unfold formal weaknesses in the definition of local ergotropy, such as the fact that it is not guaranteed to be non-increasing in time. We then introduce the concept of *extended local ergotropy* [5] by exploiting the free-evolution of the system-environment compound. At variance with the local ergotropy, the extended local ergotropy is greater by construction, is non-increasing in time, and activates the potential of work extraction in many cases. We provide examples based on the Jaynes-Cummings model, presenting practical protocols and analytic results that serve as proof of principle for the aforementioned advantages.

We finally shift our attention to a quantum battery made up of many interacting sub-systems and study the maximum extractable work via concurrent local unitary operations on each subsystem [Fig. 1(a), right panel]. We call the resulting functional *parallel ergotropy* (PE) [6] and we devise methods to computing and bounding it. This paves the way for more realistic and feasible work-extraction protocols, if compared with those that require global operations on the multipartite system. Focusing on the bipartite case, we first observe that PE outperforms work extraction via *egoistic* strategies, in which the first agent A extracts locally on its part the maximum available work and the second agent B, subsequently, extracts what is left on the other part. For the agents, this showcases the need of cooperating for an overall benefit. Notably, the corresponding parallel capacity can detect entanglement and extending the concept of PE by leveraging system’s free-time evolution allows one to saturate the gap with the ergotropy of the whole system.

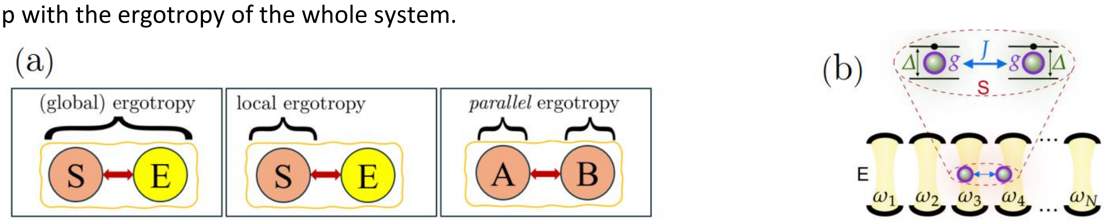

Fig. 1 (a) Schematics of (global) ergotropy: a single global unitary; local ergotropy: local unitary on S only; parallel ergotropy [6] in case of subsystems: concurrent local unitaries on subsystems A and B. The parts interact via a Hamiltonian term represented by a red double arrow. Unitaries are depicted as curly brackets and the joint AB (or SE) state by the yellow envelope. (b) Multimode two-qubit Rabi model from [4] and analyzed in Ref. [3] in connection with local ergotropy.

References

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