Berezinskii-Kosterlitz-Thouless and dynamical quantum phase transitions in the open quantum Rabi Model

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Abstract: We analyze the thermodynamic and dynamical properties of the open Rabi model and its two-qubit extension, revealing Berezinskii-Kosterlitz-Thouless thermodynamic quantum phase transition and two dynamical quantum phase transitions, highlighting their interplay in open quantum systems.

We investigate the dynamical and thermodynamic properties of the open Rabi model and its two-qubit extension, focusing on quantum phase transitions (QPTs) and dynamical quantum phase transitions (DQPTs) induced by environmental interactions [1]. To this end, we employ state-of-the-art numerical methods, including the density-matrix renormalization group (DMRG) algorithm for equilibrium states and the time-dependent variational principle (TDVP) algorithm for non-equilibrium dynamics. These methods model complex open quantum systems as matrix product states (MPSs), a 1D tensor network variational ansatz. In addition, we apply worldline Monte Carlo simulations at equilibrium and approaches *à la* Feynman and Mori for both equilibrium and dynamical properties.

On the thermodynamic side, we demonstrate that increasing the coupling to the oscillator mediating interactions with the environment induces a Berezinskii-Kosterlitz-Thouless (BKT) transition. This transition occurs in both the one- and two-qubit open Rabi models within the experimentally accessible deep-strong coupling regime. The transition is observed by exactly mapping the model onto 1D long-range ferromagnetic Ising spin chains (via path integral technique) and identifying the squared total magnetization of the mapped system as the order parameter of the transition [2].

On the other hand, out of equilibrium, we investigate the relaxation dynamics of the qubits and oscillator. At low environmental coupling, Rabi oscillations dominate, but as the coupling increases, the relaxation functions decay exponentially and eventually remain constant at the BKT transition (see Fig. 1(a)). Furthermore, we prove that, by quenching the coupling between the qubits and the oscillator, the system undergoes DQPTs [3], characterized by two distinct types of transitions with differing critical exponents [4]. These transitions are identified through kinks in the rate function of the Loschmidt echo, appearing in the same parameter range where the thermodynamic BKT transition takes place (see Fig. 1(b)). The onset of these transitions is also signaled by bimodal magnetization distributions and changes in qubits entanglement.

As an application, we explore local ergotropy, defined in literature as the maximum extractable work via a local unitary on the subsystem, without acting on its environment. We propose a realistic, tunable protocol encompassing charging, quasi-decoherence-free storage, and work extraction [5]. Our results show that strong couplings to a bath are not detrimental to work extraction and local ergotropy and its fluctuations mark the QPT.

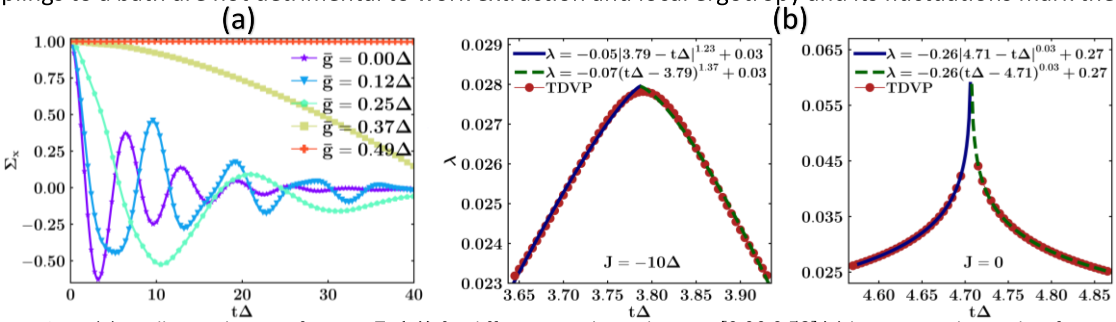

Fig. 1. (a) Oscillator relaxation function $\Sigma_x(t\Delta)$ for different coupling values $\bar{g} \in [0.00, 0.59]\Delta$ (shown in rainbow colors from violet to red) near the critical point, computed using TDVP. (b) Loschmidt echo rate function $\lambda(t\Delta)$ for $J = -10\Delta, g = 0.48\Delta$ (left) and $J = 0, g = 0.58\Delta$ (right). The red points are computed through TDVP. The fitting functions for the left branch (blue solid line) and the right branch (green dashed line) illustrate two distinct critical behaviors.

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