Shadow Quantum Linear Solver:
A Resource Efficient Quantum Algorithm for Linear Systems of Equations

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Abstract: We propose the Shadow Quantum Linear Solver, a novel quantum algorithm for solving linear systems, combining Variational Quantum Algorithms with classical shadows, requiring fewer qubits, and offering exponential advantages in circuit execution efficiency.

Solving linear systems is essential in science and technology, with the HHL algorithm offering exponential quantum speedup[1], but its practical use is limited by the constraints of current noisy intermediate-scale quantum (NISQ) hardware[2]. In this work, we propose the Shadow Quantum Linear Solver (SQLS)[3], a variational quantum algorithm[4] that leverages classical shadows[5] to solve linear systems of equations. Due to the exponential size of Hilbert space, the SQLS requires a logarithmic number of qubits compared to the linear system size. The SQLS takes as input a linear system of the form Ax=b. By assuming that both A and b are given as a linear sum of Pauli, our local cost function then requires the evaluation of a large number of expectation values of Pauli strings, which can be efficiently done with classical shadows. Once evaluated, the cost function is used to update the parameters through optimization techniques. Finally, once the cost function reaches a set threshold value, the flexibility of classical shadows allows one to use them to perform tomography.

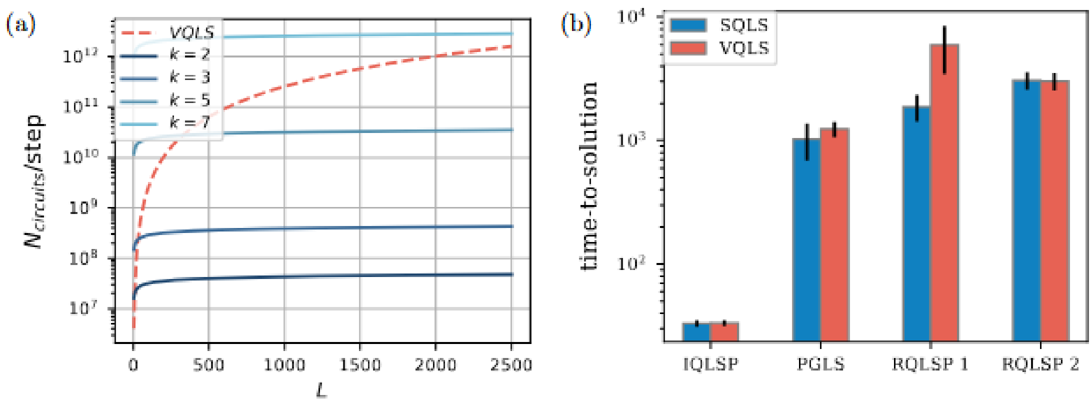

Fig. 1 The Shadow Quantum Linear Solver (SQLS) outperforms the Variational Quantum Linear Solver in resource usage and convergence times. Panel (a) compares the scaling of circuits per optimization step for a $2^{50}\times 2^{50}$ linear system, while panel (b) shows the time-to-solution, defined as the number of cost function evaluations required to achieve fidelity $1-\epsilon$ for a number of linear systems.

We perform a number of experiments in order to validate how the SQLS performs when compared to other notorious variational approaches to linear systems[6]. In the first study we look at resource usage. We find that the SQLS uses: less qubits and less controlled operations. Furthermore, we find that, for linear systems with low locality, the SQLS uses an exponentially smaller number of circuit executions. This can be seen in Fig. 1a. We also study the convergence time of the SQLS. In this case we find that our proposed methodology has similar convergence times when compared to other notorious variational algorithms for equal error. This can be seen in Fig. 1b where we test the convergence times for a number of linear systems.

Example References

[1] A. W. Harrow, A. Hassidim, and S. Lloyd, "Quantum algorithm for linear systems of equations," Physical Review Letters, vol. 103, no. 15, Oct. 2009.

[2] J. Preskill, "Quantum computing in the nisq era and beyond," Quantum, vol. 2, p. 79, Aug. 2018.

[3] Ghisoni, F., Scala, F., Bajoni, D., & Gerace, D. (2024). *Shadow Quantum Linear Solver: A Resource Efficient Quantum Algorithm for Linear Systems of Equations*. arXiv preprint arXiv:2409.08929. Retrieved from https://arxiv.org/abs/2409.08929.

[4] M. Cerezo, A. Arrasmith, R. Babbush, et al., "Variational quantum algorithms," Nature Reviews Physics, vol. 3, no. 9, pp. 625–644, Aug. 2021

[5]H.-Y. Huang, R. Kueng, and J. Preskill, "Predicting many properties of a quantum system from very few measurements," Nature Physics, vol. 16, no. 10, pp. 1050–1057, Jun. 2020.

[6]C. Bravo-Prieto, R. LaRose, M. Cerezo, Y. Subasi, L. Cincio, and P. J. Coles, "Variational quantum linear solver," Quantum, vol. 7, p. 1188, Nov. 2023