Classical simulation of bosonic-encoded quantum computations

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Abstract: We introduce a framework to classically simulate quantum circuits with a large number of highly-squeezed Gottesman-Kitaev-Preskill stabilizer states, thus offering useful benchmarks to guide the experimental design of fault-tolerant quantum computing architectures with bosonic codes.

Designing classical algorithms capable of reliably simulating quantum computations is essential for the development of quantum computing architectures. While these algorithms may not be efficient in general, they stand out as critical benchmarks for assessing experimental platforms. We focus on continuous-variable (CV) architectures which, via the usage of CV (bosonic) codes, have demonstrated significant experimental potential for protecting quantum processors from environmental errors [1]. Among these, the Gottesman-Kitaev-Preskill (GKP) code stands out as a leading candidate [2]. However, given that these states are highly non-Gaussian and their Wigner function is highly negative, standard simulation frameworks based on Gaussianity or on the positivity of traditional quasiprobability distributions cannot be applied. In this work [3], we provide a novel framework to classically simulate a universal computational model based on GKP states, whose generic circuit is represented in Fig 1 (left panel).

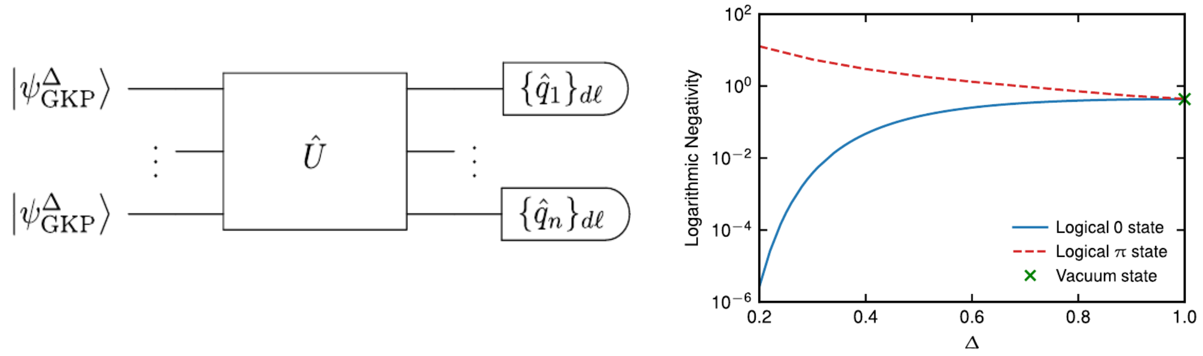

Fig. 1 (Left panel) The circuit class that we consider: the input is given by arbitrary GKP-encoded qudits, with odd dimension *d* and inverse squeezing parameter *Δ*; the evolution *U* includes all GKP-encoded Clifford operations; the measurements are modular homodyne measurements (corresponding to GKP-encoded Pauli measurements). (Right panel) Zak-Gross Wigner logarithmic negativity for the 0-logical and π-logical GKP state as a function of the inverse squeezing parameter Δ. The cross represents the negativity of the vacuum state.

We start by providing an algorithm able to *efficiently* simulate such computational model when the initial states are restricted to ideal infinitely squeezed (*Δ*=0) stabilizer GKP states. Rather than using traditional quasiprobability distributions, our approach leverages the Zak-Gross Wigner function, recently introduced in Ref. [4]. Then, we extend this algorithm to the practical setting of realistic, finitely squeezed (*Δ>0*) GKP states, encoding arbitrary logical states. The complexity of the simulation is directly linked to the negativity of the Zak-Gross Wigner function and, for stabilizer GKP inputs, it decreases with increasing squeezing (see Fig 1, right panel). By direct calculation, we show that this implies only minor simulation overhead in the large squeezing regime, in stark contrast to existing simulators which become impractical in such scenarios. For example, with stabilizer GKP states exhibiting 12 dB of squeezing (estimated to be necessary to achieve fault-tolerance with generic GKP states), our algorithm can simulate circuits with up to 1,000 modes with less than double the number of samples required for a single input mode. Therefore, our algorithm enables the simulation of some large-scale circuits in the regime relevant to experiments targeting fault-tolerant quantum computing, positioning it as a potentially valuable tool for validating bosonic platforms designed for this purpose.

References

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