An Efficient Joint Measurement Strategy for Estimating Fermionic Observables

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Abstract: We propose a scheme for estimating non-commuting fermionic observables on near-term quantum computers via a joint measurement of their noisy counterparts, achieving state-of-the-art sample complexities and improved circuit depth and gate counts in practical settings.

A task in which quantum computers may offer practical speedups over their classical counterparts is the problem of determining low energy states of molecular Hamiltonians in quantum chemistry. However, as the system size increases, one of the main performance bottlenecks is the inability to simultaneously measure the non-commuting observables which appear in the Hamiltonian's decomposition. Attempts to overcome this difficulty have involved searching for minimal groupings of the measurements into commuting subsets, or randomized measurement strategies such as classical shadows.

In this work we consider an alternative approach for estimating non-commuting fermionic observables by implementing a simple and efficient *joint* fermionic measurement, in analogy to the strategy for multi-qubit Pauli observables in Ref. [1]. In particular, we construct a joint measurement of a modified (noisy) collection of Majorana monomials of degree–$2k$ in an N mode fermionic system, whose outcome statistics can be reproduced from the (parent) measurement and an efficient classical post-processing [2,3]. To realize this measurement we use: (i) a randomization over a set of unitaries that realize products of Majorana fermion operators; (ii) a unitary, sampled at random from a constant-size set of suitably chosen fermionic Gaussian unitaries; (iii) a measurement of fermionic occupation numbers; (iv) suitable post-processing.

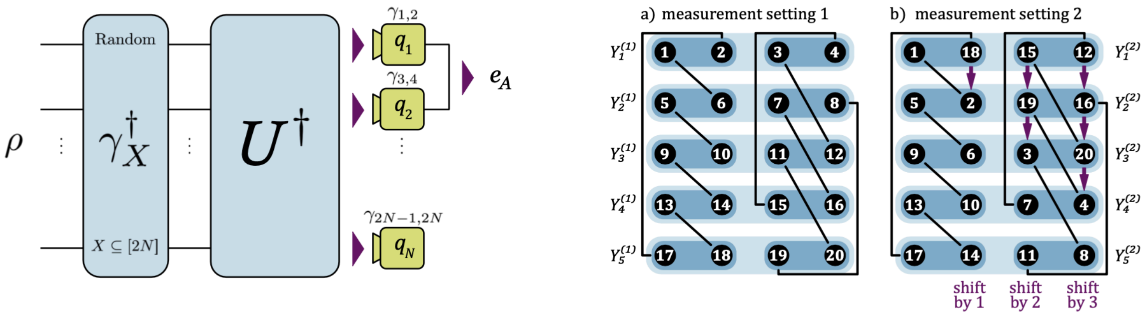

Fig. 1 Circuit implementation (left) of the joint measurement scheme described in (i)-(iv) for an N mode fermionic state. An implementation (right) of the Gaussian unitary in measurement step (ii) on a rectangular lattice of 10 qubits.

The measurement has several advantages over other methods, while ensuring the observables can be estimated with state-of-the-art sample complexity scalings (which are asymptotically optimal within the framework of joint measurability [2]). In particular, our scheme requires considerably fewer non-trivial measurement settings than previous techniques, and it is suitable for randomized error mitigation techniques due to its estimation of observables using single-qubit measurement outcomes from at most two qubits.

Our proposed implementation (Fig. 1, right) on a 2D rectangular lattice of qubits—suitable for present-day superconducting quantum computing architectures—has circuit depth $O(\sqrt{N})$ and two-qubit gate count $O(N^{3/2})$. This compares favourably with fermionic classical shadows that require depth $O(N)$ and $O(N^2)$ gates.

References

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