A Unified Framework for Shadow Tomography and Quantum Extreme Learning Machines

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Abstract: We establish strong connections between quantum shadow tomography and quantum extreme learning machines, showing that both can be framed under a single unifying metrological perspective, differing only in their prior assumptions.

We present a unifying perspective on *shadow tomography* and *quantum extreme learning machines* (QELMs), revealing that both approaches can be viewed as quantum estimation techniques that differ mainly in their assumptions regarding measurement calibration.

Shadow tomography is a powerful framework for estimating properties of quantum states without incurring the exponential overhead typically associated with full state tomography, offering strong theoretical guarantees and scalability. By encoding measurement outcomes into compact "classical shadows," this technique makes it possible to efficiently approximate many observables with rigorous error bounds. It thus provides a flexible and resource-efficient platform for characterizing diverse quantum properties—ranging from simple expectation values to more complex tasks like entanglement detection. On the other hand, QELMs and closely related quantum reservoir computing architectures avoid the need for extensive training of quantum parameters or high-fidelity control. Instead, the intrinsic dynamics of a quantum system—such as single-photon quantum walks in high-dimensional orbital angular momentum spaces—serve as a computational reservoir. This design enables robust learning of target quantum properties, including state fidelities and entanglement witnesses, with minimal experimental calibration. By only training classical post-processors directly on the measured outcomes, QELMs efficiently reconstruct expectation values of observables and other properties, remaining remarkably resilient to noise and other experimental imperfections.

To demonstrate the practical viability of these concepts, we discuss experimental implementations QELMs in a photonic system. These experiments confirm that precise knowledge of the measurement apparatus is not essential for reliable quantum state reconstruction or entanglement detection. This underscores the synergy between the theoretical underpinnings of shadow tomography and the versatility of QELM-based implementations in noisy intermediate-scale quantum platforms.

Overall, our findings highlight an avenue toward resource-efficient quantum state characterization readily adaptable to near-term quantum devices. By illustrating the interplay between standard quantum estimation principles and modern machine learning paradigms, we offer a novel framework for advancing quantum technologies through scalable and experimentally feasible approaches.

References

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