Realizing non-Hermitian dynamics in a quantum walk of structured light

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Abstract: We propose an innovative approach for simulating non-Hermitian dynamics by realizing a non-unitary photonic quantum walk based on a light beam propagating in free space and manipulated via step operators acting on its polarization and transverse momentum.

Recently, there has been a growing interest in the development of innovative platforms aimed at realizing non-Hermitian Quantum walks (n-HQWs), that involve non-unitary operators and Hamiltonians with non-Hermitian characteristics [1,2]. This paradigm opens new avenues for exploring novel quantum phenomena and applications. Here, we present a platform for the implementation of n-HQW based on light propagation through liquid-crystal plates exhibiting inhomogeneous patterns of dichroism and birefringence.

The QW experiments study the evolution of a particle conditioned by its internal degrees of freedom. This evolution occurs in the Hilbert space $H_{ext} \otimes H_{in}$, where H_{ext} is the space describing the particle's position on an infinite lattice, whose states $|m\rangle$ are denoted through integer numbers, and H_{in} is the space of internal states, described, in the 2d case, through the basis $\{|\uparrow\rangle, |\downarrow\rangle\}$.
In a Discrete-Time QW, the particle's evolution is governed by the repeated application of the single-step operator $\hat{U}$ involving two operations per step. A coin toss operation, $\widehat{W}$, that represents a spin rotation of the internal state on the Bloch sphere, and a conditional translation operation based on the internal state, acting as $\widehat{T_x}(\delta) = \cos\frac{\delta}{2}\hat{I} + i\sin\frac{\delta}{2}\Sigma[|m+1\rangle\langle m| \otimes |\downarrow\rangle\langle\uparrow| + |m-1\rangle\langle m| \otimes |\uparrow\rangle\langle\downarrow|]$, where $\hat{I}$ is the identity operator, and δ is the hopping amplitude between neighbouring sites on the lattice.

In our photonic implementation, the basis $\{|\uparrow\rangle, |\downarrow\rangle\}$ is encoded into left and right polarizations $\{|L\rangle, |R\rangle\}$, while the discretized transverse momentum of light into the lattice sites $m = (m_x, m_y)$. The particle corresponds to an optical beam propagating in free space, with its spatial mode, described as:

$$|m, \phi\rangle = A(x, y, z)\, e^{i[\Delta k_{\perp}(m_x\, x + m_y\, y) + k_z z]} \otimes |\varphi\rangle$$

where $A(x, y, z)$ is the Gaussian envelope in the $\hat{x}\hat{y}$ plane, $|\varphi\rangle$ is its polarization, $\Delta k_{\perp}$ is the transverse wavevector component, and $k_z \approx \frac{2\pi}{\lambda}$ (with λ being the light's wavelength).

During the evolution, light interacts with liquid crystal (LC) plates that perform both the coin toss and conditional translation operations, described by the Jones matrix in the LR basis as:

$$L_{LR}(\delta, \alpha) = \begin{pmatrix} \cos\frac{\delta}{2} & i\,\sin\frac{\delta}{2}\, e^{-2i\alpha} \\ i\sin\frac{\delta}{2} e^{2i\alpha} & \cos\frac{\delta}{2} \end{pmatrix}$$

where α is the LC optic axis orientation, and δ is the birefringent retardation, controlled via an external field. The coin toss and the conditional translator operators are achieved, respectively, with a uniform LC plate (α=0, δ=π/2) and using a g-plate with α=α_0+απx/Λ (Λ is the spatial periodicity) and δ=π.

Losses are implemented by doping the g-plate with a dichroic dye. So, combining the birefringence and dichroism's properties, a dichroic LC plate can be expressed as:

$$L_{LR}(\delta, \eta, \alpha) = e^{-\eta'} \begin{pmatrix} \cos\frac{\delta + i\eta}{2} & i\,\sin\frac{\delta + i\eta}{2}\, e^{-2i\alpha} \\ i\sin\frac{\delta + i\eta}{2} e^{2i\alpha} & \cos\frac{\delta + i\eta}{2} \end{pmatrix}$$

with η the dichroic parameter, and η' a global attenuation factor [3].

As the light passes through these plates, the modes combine into superpositions, forming an optical beam. In the focal plane of an imaging lens (far field), the modes are separated [4] and reflect the particle's probability distribution across lattice sites.

We implement a one-dimensional QW protocol up to 5 steps, where each step involves the repeated application of the evolution operator $\widehat{U} = \widehat{T}_X(\delta, \eta)\widehat{W}$.

The protocol is tested under various configurations, varying the input polarization (H, V, L, R) and the dichroic parameter η. Results are averaged over four independent experiments. Figure 1 shows representative distributions achieved a n-HQWs.

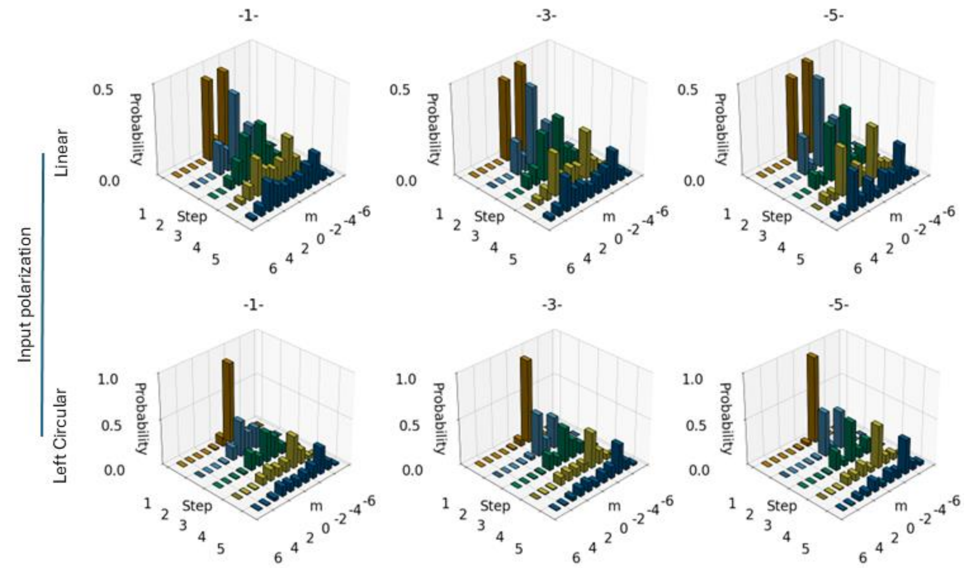

Figure 1. This figure shows the evolution of a non-Hermitian QW up to 5 steps, both for a linear (up) and a circular (down) input polarization. Along each row, the evolutions for the same value of η are shown. -1-, -3-, -5- labels are linked to different values of η.

The measured distributions align well with numerical simulations, with similarity values consistently above 0.97.

The efficacy of our model introduces it as a potent tool for exploring topological phases [1,2].

Example References

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