Quantifying the Complexity of Learning Quantum Features

Leonardo Banchi[1,2]**,Jason Pereira**[2]**, Alessandro Cuccoli**[1,2]**, Mehran Khosrojerdi**[1] **, Marco Zamboni**[1]

1. *Department of Physics and Astronomy, University of Florence, via G. Sansone 1, I-50019 Sesto Fiorentino (FI), Italy*
2. *INFN Sezione di Firenze, via G. Sansone 1, I-50019, Sesto Fiorentino (FI), Italy*

Abstract: We will present a combination of different results obtained by our group in the last few years, about quantifying the complexity of learning with quantum data, such as quantum states, quantum dynamics and quantum channels.

The ability to extract general laws from a few known examples depends on the complexity of the problem and on the amount of training data. In the quantum setting, the learner's generalization performance is further challenged by the destructive nature of quantum measurements that, together with the no-cloning theorem, limits the amount of information that can be extracted from each training sample [1].

Example applications include the classification of quantum phases of matter [2], which are encoded into ground states of quantum many-particle systems, decision problems such as learning to classify entangled vs. separable states [3], and sensing applications such as quantum-enhanced object/pattern recognition [4]. We will show how to adapt bounds from statistical learning theory [5,6] to assess which of these tasks are easy for a learner, in the sense of requiring few training data-points and few measurement settings and shots.

In some cases, the uncertainty coming from a few measurement shots can be the dominant source of errors. We have identified an instance of this possibly general issue by focusing on the classification of maximally entangled vs. separable states, showing that this toy problem becomes challenging for learners unaware of entanglement theory [3]. Our results show that the naive application of classical machine-learning methods to the quantum setting is problematic, and that a better theoretical foundation of quantum learning is required [5,6].

On the other hand, by using a combination of classical and quantum techniques, such as tensor networks, kernel methods, generalization bounds, quantum algorithms, and shadow estimators, we have shown that learning observables whose measurement allows for identification of quantum phases of matter is easy [2], as, in some cases, only a polynomial number of measurements is required. An important application of our findings is the classification of the phases of matter obtained in quantum simulators, e.g., cold atom experiments, capable of efficiently preparing ground states of complex many-particle systems and applying simple measurements, e.g., single qubit measurements, but unable to perform a universal set of gates.

Finally, we discuss the possibility of achieving quantum advantage in learning classical data, specifically temporal stochastic processes. By studying the trade-off between accuracy and memory requirements, we have shown [7] that quantum models can reach the same accuracy with less memory, or alternatively, better accuracy with the same memory. We also discuss the implications of this result for learning tasks.

References

[1] V. Gebhart, R. Santagati, A. A. Gentile, E. Gauger, D. Craig, N. Ares, L. Banchi, F. Marquardt, L. Pezze', C. Bonato, Learning Quantum Systems, Nature Reviews Physics, 5, 141-156, (2023).

[2] M. Khosrojerdi, J. Pereira, A. Cuccoli, L. Banchi, Learning to Classify Quantum Phases of Matter with a Few Measurements. Quantum Science and Technology (accepted 2024).

[3] L. Banchi, L., J. Pereira, M. Zamboni, Few measurement shots challenge generalization in learning to classify entanglement. arXiv preprint arXiv:2411.06600.

[4] L. Banchi, Q. Zhuang, S. Pirandola, Quantum-enhanced barcode decoding and pattern recognition, Physical Review Applied 14 (6), 064026, (2020)

[5] L. Banchi, J. Pereira, S. Pirandola, Generalization in quantum machine learning: a quantum information standpoint, PRX Quantum 2 (4), 040321 (2021).

[6] L. Banchi, J. L. Pereira, S. T. Jose, O. Simeone, Statistical Complexity of Quantum Learning, Adv. Quantum Technol., 2300311, (2024).

[7] L. Banchi, Accuracy vs memory advantage in the quantum simulation of stochastic processes, Mach. Learn.: Sci. Technol. 5 025036, (2024).