Exponential optimization of adiabatic quantum-state preparation

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The infidelity of a quantum state via adiabatic preparation decays exponentially with computational time. We analytically derive its characteristic time, enabling optimal Hamiltonian design, and validate this method with numerical experiments on prototypical spin models.

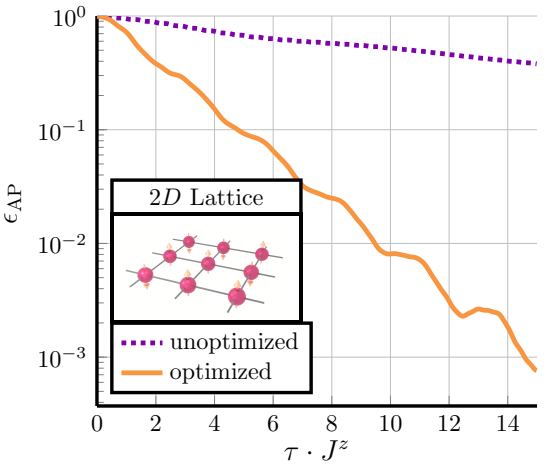

FIG. 1. Adiabatic-preparation infidelity error $\varepsilon_{\mathrm{AP}}$ as a function of the adiabatic timescale time τ for the ground state preparation of the 2- dimensional Heisenberg model. The Hamiltonian defining the system is $H = \sum_{\langle i,j \rangle} \left[J^z Z_i Z_j + J^x X_i X_j \right]$, with $\langle \cdot, \cdot \rangle$ denoting the sum over the first-neighbours sites. The naively-chosen auxiliary Hamiltonian is $H_0 = \sum_{\langle i,j \rangle} \left[J^z Z_i Z_j \right]$ The results in the plot refer to a 9-sites square lattice with a ratio $J^x/J^z = 5$ between the coupling constants and periodic boundary conditions. Source: `https://arxiv.org/abs/2405.03656`

The preparation of a given quantum state on a quantum computing register is a typically demanding operation, requiring a number of elementary gates that scales exponentially with the size of the problem. In view of performing quantum simulations of manybody systems, this limitation might severely hinder the actual application of the noisy quantum processors that are currently available. In our work (Ref. [1]) we focus on adiabatic processes to prepare quantum states [2, 3]. In addition to the Hamiltonian of the system to be simulated, the adiabatic preparation requires an auxiliary Hamiltonian H_0 that can be chosen with high arbitrariness. Our aim is to provide a theoretically guided procedure to select the optimal auxiliary Hamiltonian, i.e. the one that allows to prepare the highest-fidelity approximation of the target quantum state within a fixed depth of the quantum circuit. In Ref. [1], we theoretically derive a bound to the error in state preparation that shows an exponential scaling as a function of the adiabatic timescale τ, which is proportional to the circuit depth, and we provide an expression for its characteristic time, where the dependence from H_0 is made explicit. Therefore, the auxiliary Hamiltonian minimizing the characteristic time formula showcases an exponential suppression of the error if compared with a naively-chosen one. We perform extensive numerical experiments to test our mathematical result on typical spin-models, such as the one- and two-dimensional Ising and Heisenberg models, confirming that the exponential bound is indeed realized and observing an exponential advantage for the optimized adiabatic processes against the unoptimized one. Our results provide a promising strategy to perform quantum simulations of manybody models via Trotter evolution on near term quantum processors.

[1] D.Cugini, et al., Exponential optimization of quantum state preparation via adiabatic thermalization, arXiv preprint arXiv:2405.03656, (2024).
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