Threshold Quantum State Tomography for Qudits with Optimized Settings

Giovanni Garberoglio[1], **Daniele Binosi**[1], **Diego Maragnano**[2], **Maurizio Dapor**[1], **Marco Liscidini**[2]

1. *European Centre for Theoretical Studies in Nuclear Physics and Related Areas – Fondazione Bruno Kessler, Strada delle Tabarelle 286, I-38123 Villazzano (TN).*
2. Dipartimento di Fisica, Università di Pavia, via Bassi 6, 27100 Pavia.

Abstract: We describe Enhanced Compressive Threshold Quantum State Tomography (ECT-QST), a procedure that optimizes the number of measurement settings, enabling resource-efficient density matrix reconstruction for arbitrary qudit systems without sacrificing accuracy.

Quantum State Tomography (QST) is the procedure for reconstructing the density matrix of a quantum system by measuring a sufficiently large number of observables. In the conventional approach, known as full QST, the expectation values of at least 2^{2N} observables must be measured to determine the density matrix of a system with N qu*b*its. To reduce the number of required projective measurements, threshold Quantum State Tomography (tQST) was recently proposed for qu*b*its systems [1, 2].

tQST introduces a threshold t and focuses only on the density matrix elements r_{ij} where $|r_{ij}|\geq t$. When $t\rightarrow 0$, tQST converges to full QST, requiring 2^{2N} measurements. However, for quantum states with sparse density matrices, tQST enables density matrix reconstruction using significantly fewer projective measurements.

This work introduces a novel method called Enhanced Compressive Threshold Quantum State Tomography (ECT-QST), designed to reconstruct the density matrix of systems comprising N qu*d*its. ECT-QST integrates tQST with SU(d)-generator-based measurement settings, optimizing the number of required measurements while preserving reconstruction accuracy. Notably, it applies to quantum systems of any dimension without relying on assumptions about the quantum state. ECT-QST marks a significant advancement in Quantum State Tomography by reducing computational complexity and broadening its applicability to previously unattainable quantum systems [3].

We have developed the following: (i) a technique to associate two measurement settings with each matrix element— which extract information for the real and imaginary part of the matrix element, respectively—using only standard single-qu*d*it observables (such as Pauli operators in the case of qu*b*its); (ii) a procedure to prune the number of settings while still ensuring sufficient information to fully reconstruct the density matrix; and (iii) a priority-based sorting algorithm that enables progressively accurate density matrix reconstruction as additional settings are measured.

Using its implementation within the IBMQ framework, we demonstrated that ECT-QST achieves accurate density matrix reconstruction for systems with up to $N=7$ qu*b*its. The required number of measurement settings scales as O(1) for GHZ states, O(N^2) for W states, and O(N) for pure random states. In the case of 7-qubit systems ECT-QST requires on average 50 settings to reconstruct random states, resulting in fidelities greater than 90% with respect to the state reconstructed via full QST, which requires 2187 settings. These scaling factors are significantly smaller than the O($r2^N$) required by the original compressed sensing approach [4] for a rank-r density matrix of N qu*b*its and are comparable to the scaling achieved by the adaptive version of the compressed sensing method [5].

References

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