A Scalable Quantum Neural Network for SRBB-Based Approximate Synthesis

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Abstract: In this work, scalable quantum neural networks are introduced to approximate unitary evolutions through topological Lie algebras. Subsequently, they are optimized in terms of CNOTs and implemented, testing the approximation also on real IBM hardware.

Designing quantum circuits for unitary evolutions on a multi-qubit system and identifying suitable approximations for a given unitary is known as gate synthesis. This work exploits an algebraic approach to gate synthesis based on optimization methods [1] and Lie algebras for scalable parameterizations of unitary quantum operators [2].

A scalable quantum neural network (QNN) is proposed to approximate unitary evolutions through the Standard Recursive Block Basis (SRBB) and, subsequently, redesigned with a reduced number of CNOTs. First, a revised version of the recursive algorithm that builds the SRBB is presented, framed in the original scalability scheme already known to the literature only from a theoretical point of view [2]. 2-qubit systems are proved to be outside the original scalability scheme due to their peculiar algebraic properties. Furthermore, 3- and 4-qubit systems are described following the synopsis of a new reformulation; they represent concrete case studies, never treated in literature before, to handle the increasing complexity of the algebraic basis.

A new algorithm capable of reducing the total number of CNOTs is discovered for n>2, thus deriving a new implementable scaling scheme that requires only one single layer of approximation. From the mathematical algorithm, the scalable CNOT-reduced QNN is implemented, and its performance is assessed with a variety of different unitary matrices, both sparse and dense, up to 6 qubits via the PennyLane library.

The effectiveness of the approximation is measured with different metrics in relation to two optimizers: a gradient-based method (Adam) and the Nelder-Mead method. The tests show novel and better results when sparse matrices are used; with Nelder-Mead, the network achieves better approximations in less time than the state of the art, while Adam allows for faster approximations even in configurations that cannot be approximated using Nelder-Mead. Furthermore, novel results with dense random matrices are achieved with just a single approximation layer. The approximate SRBB-based synthesis algorithm with CNOT-reduction is also tested on real IBM hardware (Brisbane, Fez) to assess the network usability with 2 qubits and compared with other synthesis methods available in the literature. A portion of the results is shown below.

A short paper describing this work was presented at the IEEE International Conference on Quantum Computing and Engineering, in September 2024 (the proceedings of the conference have not been published yet). A more recent and largely extended version of the paper is available on the arXiv [3].

Ideal matrix	Time taken by our method + Adam opt	Time taken by our method + Nelder Mead opt	Time taken in seconds in [2]	Error from our method + Adam	Error from our method + Nelder Mead opt	Error from [2]
sparse U(4)	$8s$	$11 \sim 24s$	90	10^{-3}	$10^{-15} \sim 10^{-16}$	$10^{-12} \sim 10^{-15}$
random U(4)	6	70	–	10^{-3}	10^{-14}	–
sparse U(8)	$40s$	$< 1h$	–	10^{-2}	$10^{-10} \sim 10^{-11}$	$10^{-8} \sim 10^{-9}$
sparse U(16)	$227s$	$2 \sim 3weeks$	–	10^{-1}	10^{-9}	$10^{-8} \sim 10^{-9}$

(a)

Device	Time [s] for 1024 shots	Hellinger distance
IBM Brisbane	~ 3	~ 0.06
IBM Fez	~ 6	~ 0.07

(b)

Fig. 1: (a) shows the comparison between different methods; (b) shows the approximation of a CNOT on IBM hardware

References

[1] L. Madden and A. Simonetto, “Best approximate quantum compiling problems”, ACM Transactions on Quantum Computing 3, 2 (2022).

[2] R. S. Sarkar and B. Adhikari, “Scalable quantum circuits for n-qubit unitary matrices”, 2023 IEEE International Conference on Quantum Computing and Engineering (QCE), Vol. 1. IEEE, 1078–1088.

[3] G. Belli, M. Mordacci, M. Amoretti. 2024. A Scalable Quantum Neural Network for Approximate SRBB-Based Unitary Synthesis. arXiv:2412.03083